ARTICLE IN PRESS LAA:14198

Linear Algebra and its Applications ••• (••••) •••–•••

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Signal flow graph approach to efficient and forward stable DST algorithms ☆

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A R T I C L E   I N F O

Article history:
Received 12 August 2016
Accepted 30 May 2017
Available online xxxx
Submitted by R. Vandebril

MSC:
15A23
15B10
65F35
65F50
65T50
65Y04
65Y05
65Y20
94A12

Keywords:
Discrete sine transform
Efficient and recursive algorithms
Arithmetic complexity
Error bound and stability
Sparse and orthogonal matrices
Signal flow graphs

A B S T R A C T

In this paper, signal flow graphs are mainly addressed for the efficient and forward stable Discrete Sine Transform (DST) algorithms having sparse, scaled orthogonal, rotational, rotational-reflection, and butterfly matrices. In electrical engineering, theoretical computer science, control theory, system engineering, etc., we often use signal flow graphs as a modeling tool which interconnects the system components, or represents the realization of a system as electronic devices. The objective in this paper is to establish the connection between algebraic operations used in sparse and scaled orthogonal factorizations of DST I–IV matrices, with the signal flow graph building blocks. This paper elaborates signal flow graphs for the foundational frameworks with $n = 8$ and $n = 16$, and use these together with the digital structures to describe the DST algorithms having a $(n-1)$-point signal flow graph for DST I and n-point signal flow graphs for DST II–IV. The presented DST algorithms are completely recursive, and solely based on corresponding matrices DST I–IV. These DST algorithms have low arithmetic complexity, especially the low number of multiplications, and significant speed improvement factor as opposed to most existing algorithms. Finally, this

☆ This research was partially supported by the Faculty Innovative Research in Science and Technology funding 13265 from Embry-Riddle Aeronautical University.

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http://dx.doi.org/10.1016/j.laa.2017.05.050

Please cite this article in press as: S. M. Perera, Signal flow graph approach to efficient and forward stable DST algorithms, Linear Algebra Appl. (2017), http://dx.doi.org/10.1016/j.laa.2017.05.050

paper establishes that the presented algorithms are forward stable DST algorithms.

1. Introduction

Applications of the Fast Fourier Transform (FFT) have spread to a very diverse fields in applied mathematics and electrical engineering. Even the origin of the FFT goes back to analysis of the rotation of the Helium molecule [9]. FFT has been categorized as one of the top 10 algorithms of the computer age which had the greatest influence on the development and practice of science and engineering in the 20th century [11]. FFT is used to compute the Discrete Fourier Transform (DFT) and its inverse efficiently. The Discrete Sine Transform is a Fourier-related transform similar to the DFT, but using a purely real matrix. Some applications of the DFT are: Sine and Cosine waves of the DFT with different frequencies are used to classify the traffic monitoring sites into different seasonal patterns [41]. DST has been identified as the method which generates better results for noise estimation compared with the Discrete Cosine Transform (DCT) and the DFT [10]. The discrete fractional sine transform has been identified as the method for generating fingerprint templates with high recognition accuracy [53]. DCT, DST, and DFT approximate the Karhunen Loeve Transform (KLT) and the connection of KLT to color image compression [6,24,37,38]. DST can be used to analyze image reconstruction via signal transition through a square-optical fiber lens [51], and spectral interference and additive wideband noise on the accuracy of the normalized frequency estimator can be investigated with a discrete-time sine-wave [1], to mention a few. Along with these applications, the engagement of DCT and DST in image processing, signal processing, fingerprint enhancement, quick response code (QR code), and multimode interface can also be seen in e.g., [2,7,12,13,18,20–23,25–28,42,43,46–48].

The family of so-called discrete trigonometric transforms consists of eight versions (I–VIII) of DCT and corresponding DST and these versions appear in odd or even types, and also with respect to different Neumann and Dirichlet boundary conditions [6,29,40,43]. Although there are eight versions, depending on applications in transform coding and digital filtering of signals, we consider DCT and DST matrices as varying from type I to IV. Let us consider four orthogonal types of DST having superscripts to denote the type of DST and a subscript to denote the order of the DST in the matrix form;

$$\textit{DST-I}: \quad \mathbf{S}_{n-1}^{I} = \sqrt{\frac{2}{n}} \left[\sin \frac{(j+1)(k+1)\pi}{n} \right]_{j,\,k=0}^{n-2},$$

$$\textit{DST-II}: \quad \mathbf{S}_{n}^{II} = \sqrt{\frac{2}{n}} \left[\epsilon_n(j+1) \, \sin \frac{(j+1)(2k+1)\pi}{2n} \right]_{j,\,k=0}^{n-1}$$

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