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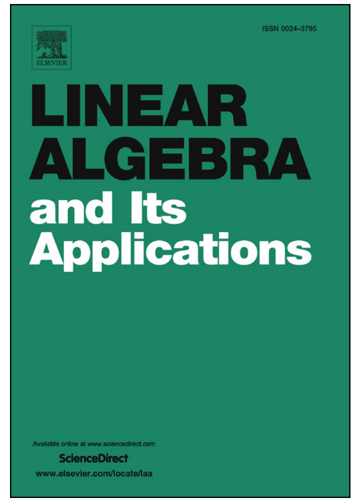

Improved random perturbation intervals of symmetric eigenvalue problem

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Improved random perturbation intervals of symmetric eigenvalue problem ☆

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Abstract

In this paper, by transforming the symmetric random perturbed matrix into a special form and using the result from [C. K. Li, R. C. Li. A note on eigenvalues of perturbed Hermitian matrices. Linear Algebra Appl. 395 (2005) 183-190], we first present an improved random perturbation interval. Under the random perturbation in the interval, the simple eigenvalue of the original matrix is still simple. Then, we extend the result to the case of multiple eigenvalues. Finally, numerical experiments are given to illustrate the obtained results.

Keywords: Symmetric eigenvalue problem, Random perturbation, Simple eigenvalue, Multiple eigenvalue

2000 MSC: 15A18, 15A42, 15A52

1. Introduction

Given the symmetric matrices A and $A(\varepsilon) = A + \varepsilon B$ whose eigenvalues are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $\lambda_1(\varepsilon) \leq \lambda_2(\varepsilon) \leq \cdots \leq \lambda_n(\varepsilon)$ respectively, where ε is a small real number and B is a fixed perturbation matrix, the aim of perturbation analysis of symmetric eigenvalue problem is to find the relationships between λ_i and $\lambda_i(\varepsilon)$ and their eigenvectors. This problem has many important applications. For instance, as an important method of clustering in graph theory, spectral clustering is based on the eigenvalues and eigenvectors of Laplacian matrix which is symmetric [8]. However, because of the noise (perturbation) of data matrix, the computed eigenvalues and eigenvectors are usually inaccurate. To make sure no significant changes appearing in the spectral clustering, we need to consider the magnitude of the perturbation of the data matrix to avoid significant changes appearing in the eigenvalues whose eigenvectors are used for clustering. For example, we hope that those eigenvalues don't swap with other eigenvalues after the perturbation [14].

There has been a long history and a lot of work on the perturbation analysis of symmetric eigenvalue problem; see the famous books [2, 13] and the recent reviews in [7, 10]. However, most of the past work on this problem considered the perturbation to be deterministic, except some work in [6, 12, 14, 15]. As discussed in [6, 15], considering the perturbation to be random is more tally with the actual situation.

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