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Differing instructional modalities and cognitive structures: Linear algebra

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A R T I C L E I N F O

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A B S T R A C T

This paper discusses the aspects of twelve first-year linear algebra students' thinking modes displayed on their interview responses to questions addressing linear independence ideas. Studying thinking modes allowed us to make inferences about the role of differing instructional modalities in shaping one's cognitive structures.

1. Introduction

This paper discusses the aspects of twelve first-year linear algebra students' thinking modes displayed on their interview responses to questions addressing linear independence ideas. Interviews were conducted right after the completion of an investigative

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assignment. Additionally, interview responses were qualitatively analyzed using a framework by Sierpinska [26].

The majority of the published work on linear algebra education discusses learning difficulties with basic linear algebra concepts. Many of these works suggest that students have problems with the high abstraction level of linear algebra materials. The high degree of formalism in linear algebra seems to make students feel a lack of connection to what they already know in mathematics. Furthermore, the axiomatic approach to linear algebra appears to give many students the feeling of learning a topic that is not necessary for their majors. Harel [16–19], for example, reinforces these claims in his statement "*understanding an algebraic system which does not have an easily accessible concrete or visual representation may result in cognitive obstacles for students*." Dorier and Sierpinska [12], furthermore, add that an understanding of linear algebra requires a fair amount of cognitive flexibility.

Another area of difficulty appears to be with multiple representational approaches. Seemingly, students have difficulty in recognizing different representations of a concept. Many lack logic and set theory knowledge [1,4,10–12,21]. Specifically, students' lack of skills in elementary Cartesian Geometry [12] and their inadequate set theory knowledge [5–9] seem to cause many of the learning difficulties in linear algebra courses.

1.1. Role of technology

Due to advances in technologies such as digital computers used widely in engineering schools [2,27] and the use of linear algebra concepts in these environments, linear and matrix algebra are among the advanced mathematics courses attracting more and more students from other disciplines [28]. These students are usually not prepared, or at best ill-prepared for the high abstraction level of matrix algebra courses. Many get sufficiently lost in the abstraction that even the simplest ideas become difficult to comprehend, thereby creating high stress leading to "burn out" and ultimately high failure rates [5–10].

According to Dubinsky [13] and Harel [16–19], students can cope with abstraction if flexibility between the multiple representations of the same concept is achieved. Moreover, one may overcome abstraction if concept images (defined as all mental pictures, properties, and processes associated with the idea) and concept definitions (described as a form of symbols used to specify the concept [20]) do not contradict one another. On the other hand, others argue that flexibility between multiple representations without inquiry may not provide the cognitive support students need in coping with abstraction [7,8,10,14,17,18,24]. Thus, many researchers suggest that technology coupled with inquiry may provide the first-hand knowledge that learners need to make better sense of the highly abstract second-hand knowledge. First-hand knowledge is defined as the knowledge obtained through direct experiences, while second-hand knowledge is the knowledge obtained from the secondary sources such as lectures and textbooks [25]. For instance, Leron and Dubinsky [23] reported that as a result of writing programs in

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