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Canonical form for the refined Wiener–Hopf equivalence relation for nonsingular 3×3 polynomial matrices

Itziar Baragaña [a,1], M. Asunción Beitia [b,1], Inmaculada de Hoyos [c,*,1]

[a] Departamento de Ciencia de la Computación e IA, Facultad de Informática, Universidad del País Vasco, UPV/EHU, Apdo. 649, 20080 Donostia-San Sebastián, Spain
[b] Departamento de Didáctica de la Matemática y de las CCEE, Facultad de Educación y Deporte, Universidad del País Vasco, UPV/EHU, 01006 Vitoria-Gasteiz, Spain
[c] Departamento de Matemática Aplicada y Estadística e IO, Facultad de Farmacia, Universidad del País Vasco, UPV/EHU, Apartado 450, 01080 Vitoria-Gasteiz, Spain

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We obtain a canonical form for the left $(2,1)$-Wiener–Hopf equivalence at infinity for polynomial matrices of order 3. This equivalence relation appears in a problem of structured perturbation when we consider polynomial matrix representations associated with controllable pairs.

* Corresponding author.
E-mail addresses: itziar.baragana@ehu.es (I. Baragaña), asuncion.beitia@ehu.es (M.A. Beitia), inmaculada.dehoyos@ehu.es (I. de Hoyos).

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Polynomial matrix representation
Canonical form

1. Introduction

In the problem of characterization of the feedback equivalence classes of the pairs $(A, [B_1 \quad B_2'])$ obtained from $(A, [B_1 \quad B_2])$ by means of small additive perturbations made on the matrix B_2 when (A, B_1) is controllable, we have obtained an equivalence relation suitable for the problem (see [2]), in such a way that if the structured perturbation problem is solved for a pair it is also solved for any pair in its equivalence class. That equivalence relation is called (n, m_1, m_2)-equivalence. In that paper we obtained a canonical form for the mentioned equivalence relation, which allowed us to solve the perturbation problem in some particular cases.

In [3], taking into account that we can associate with each controllable pair a nonsingular polynomial matrix, called polynomial matrix representation of the pair, we have defined an equivalence relation in the set of nonsingular polynomial matrices, in such a way that two pairs are (n, m_1, m_2)-equivalent if and only if their corresponding polynomial matrix representations are equivalent for this new relation. This equivalence relation is called left (m_1, m_2)-Wiener–Hopf equivalence at infinity, because it is finer than the left Wiener–Hopf equivalence at infinity. Moreover, we have found a reduced form for this refined equivalence relation.

In this paper we obtain a canonical form for the left $(2, 1)$-Wiener–Hopf equivalence at infinity.

The organization of the paper is the following one: in Section 2 we give the main notation, definitions and previous results; in Section 3 we present the reduced form for the left (m_1, m_2)-Wiener–Hopf equivalence at infinity relation and, in the particular case when $m_2 = 1$, we obtain some conditions for the elements of the matrices which relate two reduced forms; in Section 4 we obtain a canonical form for this equivalence relation in the particular case when $m_1 = 2$ and $m_2 = 1$.

2. Notation, definitions and previous results

A *partition* is a finite or infinite sequence of nonnegative integers almost all zero, $a = (a_1, a_2, \dots)$, in nonincreasing order. The *conjugate partition* of a, $\overline{a} = (\overline{a}_1, \overline{a}_2, \dots)$, is defined by $\overline{a}_k := \text{Card}\{i : a_i \geq k\}$. We define $a \cup b$ to be the partition whose components are those of a and b arranged in nonincreasing order.

We will denote by $\mathbb{F}$ an arbitrary field, by $\mathbb{F}^{m \times n}$ the set of matrices of size $m \times n$ and by $\text{Gl}_n(\mathbb{F})$ the group of invertible matrices of size $n \times n$.

For a given matrix pair $(A, B) \in \mathbb{F}^{n \times n} \times \mathbb{F}^{n \times m}$, $\mathcal{C}(A, B) = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}$ denotes the *controllability matrix* of (A, B). This pair is said to be completely controllable if $\text{rank}(\mathcal{C}(A, B)) = n$.

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