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Further remarks on the effect of multiple spectral values on the dynamics of time-delay systems. Application to the control of a mechanical system ☆

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A question of ongoing interest for linear Time-delay systems is to determine conditions on the equation parameters that guarantee the exponential stability of solutions. In recent works a new interesting property of time-delay systems was emphasized. As a matter of fact, the multiple spectral values for time-delay systems was characterized by using a Birkhoff/Vandermonde-based approach. Then, a multiplicity induced stability criteria were exhibited for reduced order systems; scalar delay-equations and a special class of second order systems. This work, further explores such a criteria

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Stability
Frequency-domain approach
Control design
Vibration control

and shows their applicability to the control of a mechanical system.

1. Introduction

In this paper, the special class of nonlinear eigenvalue problems derived from the frequency domain analysis of Time-delay systems is considered. An ongoing interest for such an eigenvalue analysis is motivated by the wide range of applications where time-delays occur. As a matter of fact, the asymptotic behavior of the solutions of this class of infinite dimensional dynamical systems is determined from the corresponding spectrum designating the set of the roots of the associated characteristic function often called *quasipolynomial*, that is a transcendental polynomial in the Laplace variable in which appear exponential terms induced by delays, see for instance [1]. The study of the zeros of such a class of entire functions [2] plays a crucial role especially in the analysis of the asymptotic stability of the zero solution associated with dynamical systems. Indeed, the zero solution is asymptotically stable if all the spectral values are in the open left-half complex plane [3]. Furthermore, the crucial effect of multiplicities (algebraic/geometric) of a given spectral values on the stability of the steady-state of the corresponding dynamical system is well known, see for instance [3]. But, such multiple roots may produce complex behaviors, for instance, in the case of multiple Hopf points' dynamics one refers the reader to the work [4]. One of the main ingredients allowing to multiple spectral values is the symmetry. Symmetries in dynamical systems often induce equivariance conditions, which may be associated with multiple spectral values. It is observed in [5] that the existence of multi-dimensional irreducible representations of the symmetry group may force a spectral value to be multiple. Also, such multiple spectral values may occur in optimization problems, see for instance [6].

Recent works by the authors [7–9] characterized multiple *Crossing Imaginary Roots* (CIR) for time-delay systems using a Birkhoff/Vandermonde-based approach. In [8] it is shown that the admissible multiplicity of the zero spectral value is bounded by the generic *Polya and Szegö bound* denoted PS_B, which is nothing but the *degree* of the corresponding quasipolynomial, see for instance [10]. In [7] it is shown that a given CIR with non vanishing frequency never reaches PS_B and a sharper bound for its admissible multiplicities is established. However, even the characterization of the multiplicity of a given complex (non real) spectral value may be carried out by the same approach, it involves hyperbolic/trigonometric *functional confluent Vandermonde matrices*. Moreover, in such a case the PS_B can never be reached. Furthermore, an example of a scalar retarded equation with two delays is studied in [7] where it is shown that the multiplicity of real spectral values may reach the PS_B. The corresponding system has some further interesting properties: (i) it is asymptotically stable, (ii) its spectral abscissa (rightmost

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