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Regular approximations of spectra of singular discrete linear Hamiltonian systems with one singular endpoint ☆

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This paper is concerned with regular approximations of spectra of singular discrete linear Hamiltonian systems with one singular endpoint. For any given self-adjoint subspace extension (SSE) of the corresponding minimal subspace, its spectrum can be approximated by eigenvalues of a sequence of induced regular SSEs, generated by the same difference expression on smaller finite intervals. It is shown that every SSE of the minimal subspace has a pure discrete spectrum, and the k-th eigenvalue of any given SSE is exactly the limit of the k-th eigenvalues of the induced regular SSEs; that is, spectral exactness holds, in the limit circle case. Furthermore, error estimates for the approximations of eigenvalues are given in this case. In addition, in the limit point and intermediate cases, spectral inclusive holds.

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1. Introduction

Consider the following discrete linear Hamiltonian system:

$$J\Delta y(t) = (P(t) + \lambda W(t))R(y)(t), \quad t \in \mathcal{I}, \tag{1.1$_\lambda$}$$

where $\mathcal{I} := \{t\}_{t=a}^{+\infty}$ is an integer interval, a is an integer; J is the $2n \times 2n$ canonical symplectic matrix, i.e.,

$$J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix},$$

with the $n \times n$ identity matrix I_n; Δ is the forward difference operator, i.e., $\Delta y(t) = y(t+1) - y(t)$; the weight function $W(t) = \text{diag}\{W_1(t), W_2(t)\}$, $W_1(t)$ and $W_2(t)$ are $n \times n$ positive semi-definite matrices; $P(t)$ is a $2n \times 2n$ Hermitian matrix; the partial right shift operator $R(y)(t) = (y_1^T(t+1), y_2^T(t))^T$ with $y(t) = (y_1^T(t), y_2^T(t))^T$ and $y_1(t)$, $y_2(t) \in \mathbf{C}^n$; λ is a complex spectral parameter.

It is evident that $P(t)$ can be blocked as

$$P(t) = \begin{pmatrix} -C(t) & A^*(t) \\ A(t) & B(t) \end{pmatrix},$$

where $A(t)$, $B(t)$, and $C(t)$ are $n \times n$ complex-valued matrices, $B(t)$ and $C(t)$ are Hermitian matrices, and $A^*(t)$ is the complex conjugate transpose of $A(t)$. Then system (1.1$_\lambda$) can be written as

$$\begin{aligned} \Delta y_1(t) &= A(t)y_1(t+1) + (B(t) + \lambda W_2(t))y_2(t), \\ \Delta y_2(t) &= (C(t) - \lambda W_1(t))y_1(t+1) - A^*(t)y_2(t), \quad t \in \mathcal{I}. \end{aligned} \tag{1.2}$$

To ensure the existence and uniqueness of the solution of any initial value problem for (1.1$_\lambda$), we always assume that

($\mathbf{A}_1$) $I_n - A(t)$ is invertible in $\mathcal{I}$.

It is known that (1.1$_\lambda$) contains the following formally self-adjoint vector difference equation of order $2m$:

$$\sum_{j=0}^{m} (-1)^j \Delta^j [p_j(t)\Delta^j z(t-j)] = \lambda w(t)z(t), \quad t \in \mathcal{I}, \tag{1.3}$$

where $w(t)$ and $p_j(t), 0 \le j \le m$, are $l \times l$ Hermitian matrices, $w(t) \ge 0$, and $p_m(t)$ is invertible in $\mathcal{I}$. The reader is referred to [28] for the details.