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A class of inverse eigenvalue problems for real symmetric banded matrices with odd bandwidth ☆

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A B S T R A C T

In this paper, we propose and discuss a class of inverse eigenvalue problems for real symmetric banded matrices with odd bandwidth. For an odd $2p + 1$ with a positive integer p, the problem is to construct an $n \times n$ real symmetric banded matrix with bandwidth $2p + 1$ whose $m \times m$ leading principal submatrix is a given $m \times m$ real symmetric banded matrix with bandwidth $2p + 1$ and spectrum is a given set of real numbers $\{\lambda_i\}_{i=1}^n$, where the number of distinct real numbers of $\{\lambda_i\}_{i=1}^n$ is $2k$ when $m = pk$, or $2k+1$ when $pk < m < p(k+1)$, where m, n and k are positive integers and $m < n$. We point out that the well-known double dimensional (DD) problem is a special case of our proposed inverse eigenvalue problems. The necessary and sufficient condition for the solvability of the above inverse eigenvalue problem is derived, and the target real symmetric banded matrix can be constructed by the block Lanczos algorithms when the inverse eigenvalue problem is solvable. Several numerical examples show that our algorithms are feasible. Some concluding remarks are introduced.

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1. Introduction

An $m \times m$ matrix D_m is called a real symmetric banded matrix with bandwidth $2p+1$, where m, p are two positive integers, if it is of the form

$$D_m = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1,p+1} & & & & \\ d_{12} & d_{22} & d_{23} & \cdots & d_{2,p+2} & & & \\ \vdots & d_{23} & d_{33} & d_{34} & \cdots & \ddots & & \\ d_{1,p+1} & \vdots & \ddots & \ddots & \ddots & \cdots & \ddots & \\ & \ddots & \vdots & \ddots & d_{m-p,m-p} & \ddots & \cdots & d_{m-p,m} \\ & & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & \ddots & \vdots & d_{m-2,m-1} & d_{m-1,m-1} & d_{m-1,m} \\ & & & & d_{m-p,m} & \cdots & d_{m-1,m} & d_{m,m} \end{pmatrix}_{m\times m}, \tag{1.1}$$

where d_{ij}, $i, j = 1, 2, \cdots, m$, $|i - j| \leq p$ are real. Throughout this paper, we always assume that $d_{i,i+p}$, $i = 1, 2, \cdots, m - p$ are positive. In particular, when $p = 2$, the banded matrix (1.1) is a pentadiagonal matrix which has the following form

$$D_m = \begin{pmatrix} d_{11} & d_{12} & d_{13} & & & & \\ d_{12} & d_{22} & d_{23} & d_{24} & & & \\ d_{13} & d_{23} & d_{33} & d_{34} & d_{35} & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & d_{m-4,m-2} & d_{m-3,m-2} & d_{m-2,m-2} & d_{m-2,m-1} & d_{m-2,m} \\ & & & d_{m-3,m-1} & d_{m-2,m-1} & d_{m-1,m-1} & d_{m-1,m} \\ & & & & d_{m-2,m} & d_{m-1,m} & d_{m,m} \end{pmatrix}_{m\times m}, \tag{1.2}$$

where d_{ij}, $i, j = 1, 2, \cdots, m$, $|i - j| \leq 2$ are real. Likewise, we assume that $d_{i,i+2}$, $i = 1, 2, \cdots, m - 2$ are positive. The real symmetric pentadiagonal matrices arise in many applications, for example, vibration problem of a cantilever beam and vibration modes of the Euler–Bernoulli beam, see [1–5].

We consider a beam of length L characterized by a variable flexural rigidity EI (where E is modulus of elasticity and I is moment of inertia of cross-section) and a variable density ρ. Assume that the left end of the beam is clamped, then the classic equation governing the infinitesimal oscillations of this beam is