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ON ISOMETRY GROUPS OF SELF-ADJOINT TRACELESS AND SKEW-SYMMETRIC MATRICES

MARCELL GAÁL AND ROBERT M. GURALNICK

ABSTRACT. This note is concerned with isometries on the spaces of self-adjoint traceless matrices. We compute the group of isometries with respect to any unitary similarity invariant norm. This completes and extends the result of Nagy on Schatten p -norm isometries. Furthermore, we point out that our proof techniques could be applied to obtain an old result concerning isometries on skew-symmetric matrices.

1. INTRODUCTION

Let us denote H_n^0 the real vector space of n by n self-adjoint traceless matrices which has dimension $n^2 - 1$. To exclude trivial considerations we always assume that $n \geq 2$. Let $PSU(n)$ denote the image of $SU(n)$ in $GL(n^2 - 1, \mathbb{R})$ acting on H_n^0 via the adjoint representation

$$Ad : SU(n) \rightarrow GL(n^2 - 1, \mathbb{R}), \quad A \mapsto UAU^{-1}$$

which is isomorphic to $SU(n)/\{\zeta I\}$ where ζ runs through the set of n -th roots of unity and I is the identity matrix. The symbol σ stands for the Cartan involution which sends any element of H_n^0 to its negative transpose.

In the paper [12] titled "*Isometries of the spaces of self-adjoint traceless operators*" Nagy by means of "the invariance of domain" proved that the isometries on H_n^0 are automatically surjective and thus linear up to a translation, by the Mazur-Ulam theorem. Furthermore, the complete description of the structure of linear isometries with respect to a large class of unitary similarity invariant norm, more precisely, with respect to any Schatten p -norm $\|\cdot\|_p$ was given whenever $n \neq 3$. Assume for a moment that $n > 2$. Then the result of Nagy could be reformulated as follows. If \mathcal{K} is the isometry group of $\|\cdot\|_p$ on H_n^0 , then one of the following happens:

- (i) $p \neq 2$ and \mathcal{K} is generated by $PSU(n)$, \mathbb{Z}_2 and the Cartan involution;
- (ii) $p = 2$ and $\mathcal{K} = O(n^2 - 1, \mathbb{R})$.

Here, we consider the group \mathbb{Z}_2 as operators acting on H_n^0 by scalar multiplications with modulus one. It was also pointed out in [12] that the result remains true when any unitary invariant norm is considered in the case where $n = 2$. In such a case both of the groups described above are the same.

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