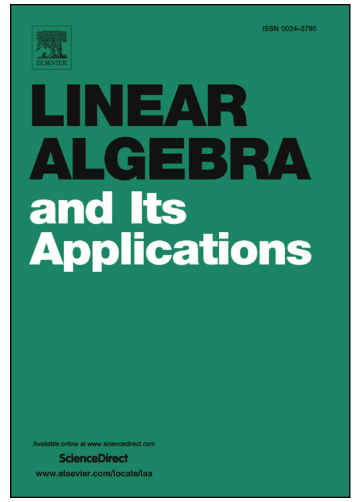
Accepted Manuscript



On isometry groups of self-adjoint traceless and skew-symmetric matrices

Marcell Gaál, Robert M. Guralnick

PII: S0024-3795(17)30677-8
DOI: https://doi.org/10.1016/j.laa.2017.12.007
Reference: LAA 14408

To appear in: *Linear Algebra and its Applications*

Received date: 13 September 2017
Accepted date: 11 December 2017

Please cite this article in press as: M. Gaál, R.M. Guralnick, On isometry groups of self-adjoint traceless and skew-symmetric matrices, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.12.007

ON ISOMETRY GROUPS OF SELF-ADJOINT TRACELESS AND SKEW-SYMMETRIC MATRICES

MARCELL GAÁL AND ROBERT M. GURALNICK

Abstract. This note is concerned with isometries on the spaces of self-adjoint traceless matrices. We compute the group of isometries with respect to any unitary similarity invariant norm. This completes and extends the result of Nagy on Schatten p-norm isometries. Furthermore, we point out that our proof techniques could be applied to obtain an old result concerning isometries on skew-symmetric matrices.

1. Introduction

Let us denote H_n^0 the real vector space of n by n self-adjoint traceless matrices which has dimension n^2-1. To exclude trivial considerations we always assume that $n \geq 2$. Let $PSU(n)$ denote the image of $SU(n)$ in $GL(n^2-1,\mathbb{R})$ acting on H_n^0 via the adjoint representation

$$Ad : SU(n) \to GL(n^2-1,\mathbb{R}), \quad A \mapsto UAU^{-1}$$

which is isomorphic to $SU(n)/\{\zeta I\}$ where ζ runs through the set of n-th roots of unity and I is the identity matrix. The symbol σ stands for the Cartan involution which sends any element of H_n^0 to its negative transpose.

In the paper [12] titled *"Isometries of the spaces of self-adjoint traceless operators"* Nagy by means of "the invariance of domain" proved that the isometries on H_n^0 are automatically surjective and thus linear up to a translation, by the Mazur-Ulam theorem. Furthermore, the complete description of the structure of linear isometries with respect to a large class of unitary similarity invariant norm, more precisely, with respect to any Schatten p-norm $\|.\|_p$ was given whenever $n \neq 3$. Assume for a moment that $n > 2$. Then the result of Nagy could be reformulated as follows. If $\mathcal{K}$ is the isometry group of $\|.\|_p$ on H_n^0, then one of the following happens:

(i) $p \neq 2$ and $\mathcal{K}$ is generated by $PSU(n)$, $\mathbb{Z}_2$ and the Cartan involution;
(ii) $p = 2$ and $\mathcal{K} = O(n^2-1,\mathbb{R})$.

Here, we consider the group $\mathbb{Z}_2$ as operators acting on H_n^0 by scalar multiplications with modulus one. It was also pointed out in [12] that the result remains true when any unitary invariant norm is considered in the case where $n = 2$. In such a case both of the groups described above are the same.

2010 *Mathematics Subject Classification.* Primary: 15A86, 47B49.
Key words and phrases. Self-adjoint traceless matrices, isometries, overgroups.