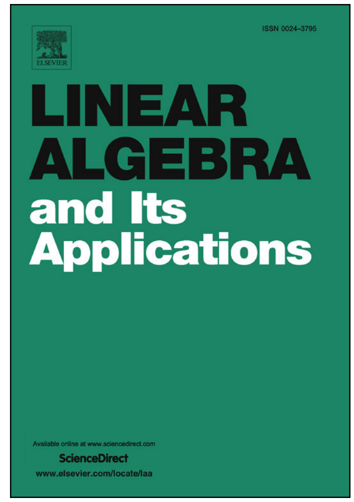
Accepted Manuscript



Random perturbation of low rank matrices: Improving classical bounds

Sean O'Rourke, Van Vu, Ke Wang

PII: S0024-3795(17)30642-0
DOI: https://doi.org/10.1016/j.laa.2017.11.014
Reference: LAA 14381

To appear in: *Linear Algebra and its Applications*

Received date: 13 May 2016
Accepted date: 15 November 2017

Please cite this article in press as: S. O'Rourke et al., Random perturbation of low rank matrices: Improving classical bounds, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.11.014

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RANDOM PERTURBATION OF LOW RANK MATRICES: IMPROVING CLASSICAL BOUNDS

SEAN O'ROURKE, VAN VU, AND KE WANG

ABSTRACT. Matrix perturbation inequalities, such as Weyl's theorem (concerning the singular values) and the Davis-Kahan theorem (concerning the singular vectors), play essential roles in quantitative science; in particular, these bounds have found application in data analysis as well as related areas of engineering and computer science.

In many situations, the perturbation is assumed to be random, and the original matrix has certain structural properties (such as having low rank). We show that, in this scenario, classical perturbation results, such as Weyl and Davis-Kahan, can be improved significantly. We believe many of our new bounds are close to optimal and also discuss some applications.

1. INTRODUCTION

The singular value decomposition of a real $m \times n$ matrix A is a factorization of the form $A = U\Sigma V^{\mathrm{T}}$, where U is a $m \times m$ orthogonal matrix, Σ is a $m \times n$ rectangular diagonal matrix with non-negative real numbers on the diagonal, and V^{T} is an $n \times n$ orthogonal matrix. The diagonal entries of Σ are known as the *singular values* of A. The m columns of U are the *left-singular vectors* of A, while the n columns of V are the *right-singular vectors* of A. If A is symmetric, the singular values are given by the absolute value of the eigenvalues, and the singular vectors can be expressed in terms of the eigenvectors of A. Here, and in the sequel, whenever we write *singular vectors*, the reader is free to interpret this as left-singular vectors or right-singular vectors provided the same choice is made throughout the paper.

An important problem in statistics and numerical analysis is to compute the first k singular values and vectors of an $m \times n$ matrix A. In particular, the largest few singular values and corresponding singular vectors are typically the most important. Among others, this problem lies at the heart of Principal Component Analysis (PCA), which has a very wide range of applications (for many examples, see [27, 35] and the references therein) and in the closely related low rank approximation procedure often used in theoretical computer science and combinatorics. In application, the dimensions m and n are typically large and k is small, often a fixed constant.

1.1. The perturbation problem.

1.1. The perturbation problem. A problem of fundamental importance in quantitative science (including pure and applied mathematics, statistics, engineering, and computer science) is to estimate how a small perturbation to the data effects

2010 *Mathematics Subject Classification.* 65F15 and 15A42.

Key words and phrases. Singular values, singular vectors, singular value decomposition, random perturbation, random matrix.

S. O'Rourke is supported by grant AFOSAR-FA-9550-12-1-0083.

V. Vu is supported by research grants DMS-0901216 and AFOSAR-FA-9550-09-1-0167.

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