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Interval matrices: Regularity generates singularity

Jiri Rohn [a,1], Sergey P. Shary [b,*]

[a] *Institute of Computer Science, Czech Academy of Sciences, Prague, Czech Republic*
[b] *Institute of Computational Technologies SB RAS and Novosibirsk State University, Novosibirsk, Russia*

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ABSTRACT

It is proved that regularity of an interval matrix implies singularity of four related interval matrices. The result is used to prove that for each nonsingular point matrix A, either A or A^{-1} can be brought to a singular matrix by perturbing only the diagonal entries by an amount of at most 1 each. As a consequence, the notion of a diagonally singularizable matrix is introduced.

* Corresponding author.
E-mail addresses: rohn@cs.cas.cz (J. Rohn), shary@ict.nsc.ru (S.P. Shary).
URLs: http://uivtx.cs.cas.cz/~rohn (J. Rohn), http://www.nsc.ru/interval/shary (S.P. Shary).
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1. Introduction and notation

Throughout this paper matrix and vector inequalities, as well as the absolute value, are understood entrywise. Also, intervals and other interval objects are denoted by bold letters in accordance with the informal standard [1].

As is well known, a square interval matrix

$$\boldsymbol{A} = [A - D,\ A + D] = \{\, B \mid |B - A| \le D \,\},$$

where $D \ge 0$, is called *singular* if it contains a singular matrix, and it is said to be *regular* otherwise. Regularity/singularity is an important concept in the classical matrix theory, and this is true for interval matrices too. Interested readers can get acquainted with the details of research on this topic in the works [2–6] as well as in the surveys [7–9]. Some generalizations to rectangular interval matrices are given in [10].

Common sense dictates that singularity and regularity exclude each other. Yet in this paper we are going to show that regularity of an interval matrix implies singularity of four interval matrices constructed from it in a nontrivial way (we add the word "nontrivial" to emphasize that we do not take into account interval matrices like $[A - A,\ D - D]$ that are trivially singular). For the proof of these results we need four auxiliary theorems that are seemingly not generally known and that are listed in Section 2. Our main result is then formulated in Theorem 5 which says that regularity of $\boldsymbol{A}$ implies singularity of four interval matrices

$$\begin{gathered}
[D - |A|,\ D + |A|],\\
[A^{-1}D - I,\ A^{-1}D + I],\\
[DA^{-1} - I,\ DA^{-1} + I],\\
[A^{-1} - |D^{-1}|,\ A^{-1} + |D^{-1}|],
\end{gathered}$$

I being the identity matrix. From this result, we draw in Section 4 a purely linear algebraic (i.e., non-interval) consequence: for each nonsingular square matrix A either there exists a singular matrix S_1 satisfying

$$|A - S_1| \le I,$$

or there exists a singular matrix S_2 satisfying

$$|A^{-1} - S_2| \le I.$$

In addition, we introduce the concept of *diagonally singularizable matrices* and give its practical motivations. Last Section 5 brings some examples.