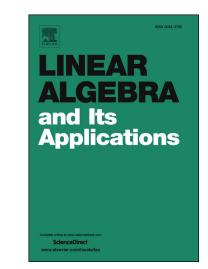
Accepted Manuscript

When does Ando-Hiai inequality hold?

Shuhei Wada



PII:S0024-3795(17)30658-4DOI:https://doi.org/10.1016/j.laa.2017.11.030Reference:LAA 14397To appear in:Linear Algebra and its Applications

Received date:15 August 2017Accepted date:28 November 2017

Please cite this article in press as: S. Wada, When does Ando–Hiai inequality hold?, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.11.030

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

WHEN DOES ANDO-HIAI INEQUALITY HOLD?

SHUHEI WADA

ABSTRACT. Let α be in (0, 1) and r > 0 and $\#_{\alpha}$ stand for the weighted operator geometric mean. We consider the following statement:

$$A, B > 0, A \#_{\alpha} B \ge I \Rightarrow A^r \#_{\alpha} B^r \ge I.$$

Ando and Hiai show that if $r \geq 1$, then this holds. In the present paper, we first prove the converse of this result, namely, the above statement holds only if $r \geq 1$. We next show that for each nonnegative continuous function f on $[0, \infty)$ with $f \leq t^r$ and $f \neq t^r$, there exist A, B > 0 such that $A \#_{\alpha} B \geq I$ and $f(A) \#_{\alpha} f(B) \geq I$. We also try to find a characterization of a continuous function fsatisfying

$$A, B > 0, A \#_{\alpha} B \ge I \Rightarrow f(A) \#_{\alpha} f(B) \ge I.$$

1. INTRODUCTION

Let \mathcal{H} be a Hilbert space with an inner product $\langle \cdot | \cdot \rangle$ and let $B(\mathcal{H})$ be the set of bounded linear operators on \mathcal{H} . A bounded linear operator A is said to be positive (denoted by $A \ge 0$) if $\langle Ax | x \rangle \ge 0$ for all $x \in \mathcal{H}$. If a positive operator A is invertible, we denote by A > 0. The set of positive operators on \mathcal{H} is denoted by $B(\mathcal{H})_+$.

A continuous real function f from $[0, \infty)$ is said to be operator monotone on $[0, \infty)$ if for two positive operators A and B, the inequality $A \ge B$ implies $f(A) \ge f(B)$. It is known that a non-negative operator monotone function f has the following property: for every $A, B \ge 0$, $f(\frac{A+B}{2}) \ge \frac{1}{2}f(A) + \frac{1}{2}f(B)$. Such a function is referred to as an operator concave function. A real function f, such that -f is operator concave, is called operator convex. It is known that every non-constant nonnegative operator convex function f on $[0, \infty)$ with f(0) = 0 can be written as f(t) = th(t) for some operator monotone function h, so the adjoint of f, denoted by $f^*(t) \left(:=\frac{1}{f(\frac{1}{t})}\right)$ is also operator convex [2].

²⁰¹⁰ Mathematics Subject Classification. Primary 47A63; Secondary 47A64.

Key words and phrases. operator mean, Ando-Hiai inequality, operator monotone.

Download English Version:

https://daneshyari.com/en/article/8898035

Download Persian Version:

https://daneshyari.com/article/8898035

Daneshyari.com