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When does Ando–Hiai inequality hold?

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WHEN DOES ANDO–HIAI INEQUALITY HOLD?

SHUHEI WADA

ABSTRACT. Let α be in $(0,1)$ and $r > 0$ and $\#_\alpha$ stand for the weighted operator geometric mean. We consider the following statement:

$$A, B > 0, A\#_\alpha B \geq I \Rightarrow A^r \#_\alpha B^r \geq I.$$

Ando and Hiai show that if $r \geq 1$, then this holds. In the present paper, we first prove the converse of this result, namely, the above statement holds only if $r \geq 1$. We next show that for each non-negative continuous function f on $[0, \infty)$ with $f \leq t^r$ and $f \neq t^r$, there exist $A, B > 0$ such that $A\#_\alpha B \geq I$ and $f(A)\#_\alpha f(B) \not\geq I$. We also try to find a characterization of a continuous function f satisfying

$$A, B > 0, A\#_\alpha B \geq I \Rightarrow f(A)\#_\alpha f(B) \geq I.$$

1. INTRODUCTION

Let \mathcal{H} be a Hilbert space with an inner product $\langle \cdot | \cdot \rangle$ and let $B(\mathcal{H})$ be the set of bounded linear operators on \mathcal{H} . A bounded linear operator A is said to be positive (denoted by $A \geq 0$) if $\langle Ax | x \rangle \geq 0$ for all $x \in \mathcal{H}$. If a positive operator A is invertible, we denote by $A > 0$. The set of positive operators on \mathcal{H} is denoted by $B(\mathcal{H})_+$.

A continuous real function f from $[0, \infty)$ is said to be operator monotone on $[0, \infty)$ if for two positive operators A and B , the inequality $A \geq B$ implies $f(A) \geq f(B)$. It is known that a non-negative operator monotone function f has the following property: for every $A, B \geq 0$, $f(\frac{A+B}{2}) \geq \frac{1}{2}f(A) + \frac{1}{2}f(B)$. Such a function is referred to as an operator concave function. A real function f , such that $-f$ is operator concave, is called operator convex. It is known that every non-constant non-negative operator convex function f on $[0, \infty)$ with $f(0) = 0$ can be written as $f(t) = th(t)$ for some operator monotone function h , so the adjoint of f , denoted by $f^*(t) \left(:= \frac{1}{f(\frac{1}{t})} \right)$ is also operator convex [2].

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