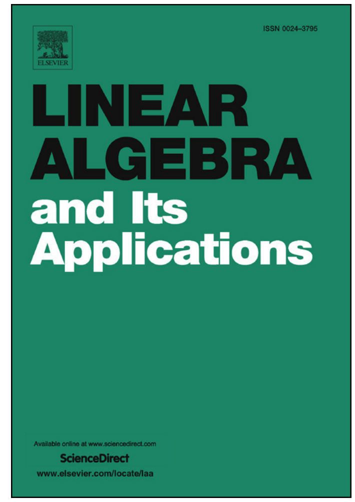

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ROBERTS ORTHOGONALITY AND DAVIS–WIELANDT SHELL

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ABSTRACT. Let $\mathcal{A}$ be a unital C^*-algebra with the unit e. We consider the elements $a \in \mathcal{A}$ which are Roberts orthogonal to the unit e. We obtain a characterization of this orthogonality in terms of the Davis–Wielandt shell of a and show that, for certain classes of elements of $\mathcal{A}$, the Roberts orthogonality of a and e is equivalent to the symmetry of the numerical range of a with respect to the origin.

1. Introduction and Preliminaries

Two elements of an inner product space are said to be orthogonal if their inner product is zero. There are many different ways how one can extend this notion to normed linear spaces (see e.g. [1, 2, 9, 18, 19, 20, 26], see also [3, 4, 5, 7, 8]). One of them is the Roberts orthogonality [26]: we say that two elements x and y of a complex normed linear space X are *Roberts orthogonal*, and we write $x \perp_R y$, if

$$\|x + \lambda y\| = \|x - \lambda y\|, \quad \forall \lambda \in \mathbb{C}. \tag{1}$$

In this paper, we study the special case of Roberts orthogonality; namely, we describe the case $a \perp_R e$, where a is an element of a unital C^*-algebra $\mathcal{A}$ and e is its unit. It turns out that this orthogonality is strongly related to a certain geometrical property of the Davis–Wielandt shell of the element a and, moreover, for certain classes of elements of $\mathcal{A}$, it can be completely described in terms of their numerical ranges.

Before stating our results, we introduce some notation and definitions we shall need in the sequel. When $\mathcal{S}$ is a subset of $\mathbb{C}^n$, we denote by $\overline{\mathcal{S}}$ the topological closure of $\mathcal{S}$, and by $\mathrm{conv}(\mathcal{S})$ the convex hull of the set $\mathcal{S}$. $\mathcal{A}$ denotes a unital C^*-algebra with the unit e. For an element a of $\mathcal{A}$, we denote by

$$\mathrm{Re}\, a = \frac{1}{2}(a + a^*), \quad \mathrm{Im}\, a = \frac{1}{2i}(a - a^*)$$

the *real* and the *imaginary part* of a.

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