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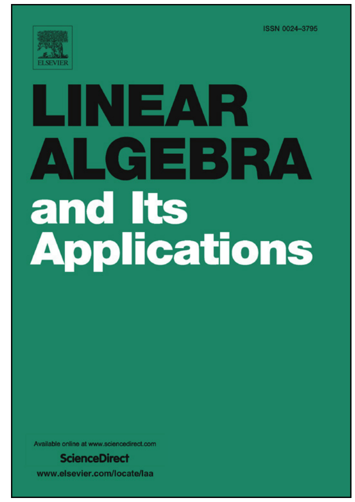
Xinlei Wang, Dein Wong, Fenglei Tian



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Signed graphs with cut points whose positive inertia indexes are two *

Xinlei Wang, Dein Wong,[†] Fenglei Tian

Department of Mathematics, China University of Mining and Technology, Xuzhou, Jiangsu, 221116, China.

Abstract: A signed graph G^σ consists of an underlying graph G and a sign function σ, which assigns each edge uv of G a sign $\sigma(uv)$, either positive or negative. The adjacency matrix of G^σ is defined as $A(G^\sigma) = (a^\sigma_{u,v})$ with $a^\sigma_{u,v} = \sigma(uv)a_{u,v}$, where $a_{u,v} = 1$ if $u, v \in V(G)$ are adjacent, and $a_{u,v} = 0$ otherwise. The positive inertia index of G^σ, written as $p(G^\sigma)$, is defined to be the number of positive eigenvalues of $A(G^\sigma)$. Recently, Yu et al. ([12], Elect. J. Linear Algebra, 31 (2016): 232-243) characterized the signed graphs G^σ with pendant vertices such that $p(G^\sigma) = 2$. In this paper, we extend the above work to a more general case, characterizing the signed graphs G^σ with cut points whose positive inertia index is 2.

Keywords: positive inertia index; signed graphs; adjacency matrix

AMS classification: 05C05; 05C50

1 Introduction

Throughout the paper, we consider signed simple graphs. Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. The adjacency matrix $A(G) = (a_{ij})_{n\times n}$ of G is an $n \times n$ matrix whose (i, j)-entry a_{ij} is 1 if there exists an edge joining v_i and v_j, and $a_{ij} = 0$ otherwise. The positive inertia index of G, denoted by $p(G)$, is the number of the positive eigenvalues of $A(G)$.

Signed graphs were introduced by Harary in connection with the study of the theory of social balance in social psychology. The main connection between these graphs and the theory of social balance will not be repeated here since this paper is concerned only with theoretical results. We refer the reader to [1] to find more details. A signed graph $\Gamma = G^\sigma$ consists of an underlying graph G and a function $\sigma : E(G) \to \{+, -\}$, assigning each edge of G a sign, either positive or negative. The adjacency matrix of G^σ is defined as $A(G^\sigma) = (a^\sigma_{i,j})$ with $a^\sigma_{i,j} = \sigma(v_iv_j)a_{ij}$, where (a_{ij}) is the adjacency matrix of the underlying graph G. The positive inertia index of G^σ, written as $p(G^\sigma)$, is defined to be the number of positive eigenvalues of $A(G^\sigma)$. The nullity of G^σ,

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[†]Corresponding author. E-mail address: wongdein@163.com. Supported by "the National Natural Science Foundation of China (No.11571360)".