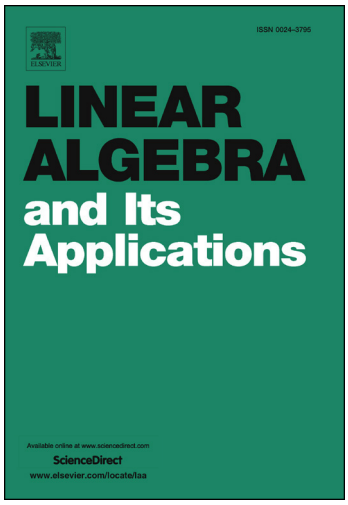

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Wold-type decompositions in Baer $*$-rings

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Abstract

A fundamental theorem due to Wold asserts that an isometry T on a Hilbert space $\mathcal{H}$ is the orthogonal direct sum of a unitary operator and a unilateral shift. Our main goal is to prove algebraic analogs of this result and its generalizations for commuting pairs of isometries, in the context of Baer*-rings. Our results not only extend Wold-type decomposition theorems from $\mathcal{B}(\mathcal{H})$ to arbitrary Baer $*$-rings, but also provide purely algebraic proofs of these known decomposition theorems, which unlike their classic analogs are independent of the underlying Hilbert space.

Keywords: Baer $*$-ring; isometry; commuting isometries; Wold decomposition; unilateral shift
2010 MSC: 47A05,47A45,16W10

1. Introduction

Decomposition of a bounded operator into a direct sum of 'nicer' operators is a fundamental general question in functional analysis, as those parts carry more precise information of the given operator and are easier to deal with. The canonical Wold decomposition is one of the major results of this type which was originally introduced by Wold [23] as a decomposition of stationary processes. von Neumann's formulation of Wold's result asserts that an isometry on a Hilbert space is the orthogonal direct sum of a unitary and

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