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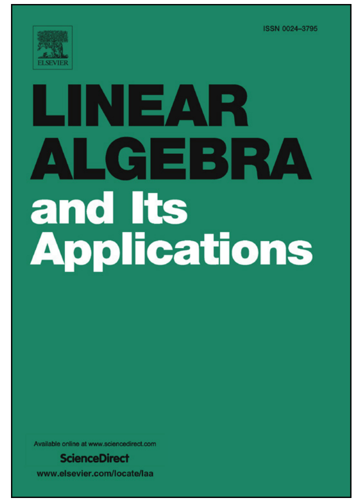

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"WRONG" SIDE INTERPOLATION BY POSITIVE REAL RATIONAL FUNCTIONS

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Abstract. Using polynomial interpolation, along with structural properties of the family of rational positive real functions, we here show that a set of m nodes in the open *left* half of the complex plane, can always be mapped to anywhere in the complex plane by rational positive real functions whose degree is at most m. Moreover we introduce an *easy-to-find* parametrization in $\mathbb{R}^{2m+3}$ of a large subset of these interpolating functions.

1. Introduction

Problem Formulation

A framework for many classical interpolation problems is as follows. Given a set of distinct nodes

$$X = \{x_1, \ \dots \ , \ x_m\}$$

and image points

$$Y = \{y_1, \ \dots \ , \ y_m\}$$

(not necessarily distinct) and a family of functions $\mathcal{F}$, find whether there exist functions $f \in \mathcal{F}$ so that

$$y_j = f(x_j) \qquad j = 1, \ \dots \ , \ m. \tag{1.1}$$

If yes, parameterize all of them, preferably within a degree bound. There is a vast literature on the subject see e.g. [2]-[7], [10], [11], [13], [15]-[19]. To simplify the discussion, we here focus on scalar real rational functions. Thus degree simply means the maximum between the degree of the numerator and of the denominator polynomials. The polynomial (a.k.a. the Lagrange) interpolation is probably the best known problem in this framework, see e.g. [9, Section 2.10], [15] (in [16] it is attributed to [18]), for matrical version, see e.g. [12, Subsection 6.1.14]. For the case where $\mathcal{F}$ is the set of rational functions see [3] and if in addition all functions in $\mathcal{F}$ are analytic in a disk, centered at the

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