

# Accepted Manuscript

A Note on the Convexity of the Moore–Penrose Inverse

Kenneth Nordström

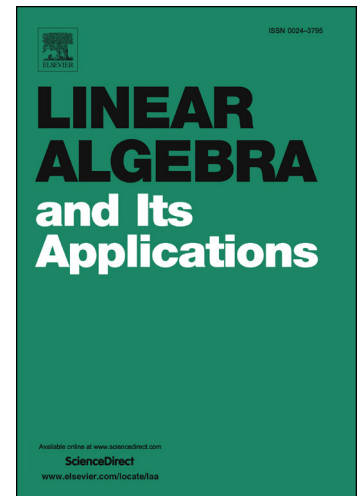
PII: S0024-3795(17)30595-5  
DOI: <https://doi.org/10.1016/j.laa.2017.10.016>  
Reference: LAA 14357

To appear in: *Linear Algebra and its Applications*

Received date: 10 May 2017  
Accepted date: 15 October 2017

Please cite this article in press as: K. Nordström, A Note on the Convexity of the Moore–Penrose Inverse, *Linear Algebra Appl.* (2018), <https://doi.org/10.1016/j.laa.2017.10.016>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



## A NOTE ON THE CONVEXITY OF THE MOORE–PENROSE INVERSE

KENNETH NORDSTRÖM

ABSTRACT. This note is a sequel to an earlier study (Nordström [7]) on convexity properties of the inverse and Moore–Penrose inverse, in which the following question was raised. Given nonnegative definite matrices  $A$  and  $B$  with Moore–Penrose inverses  $A^+$  and  $B^+$ , respectively, can one show that

$$(\lambda A + \bar{\lambda} B)^+ \leq \lambda A^+ + \bar{\lambda} B^+$$

holding for a single  $\lambda \in ]0, 1[$  is enough to guarantee its validity for all  $\lambda \in ]0, 1[$ ? [The ordering above is the partial ordering, induced by the convex cone of nonnegative definite matrices, and  $\bar{\lambda} := 1 - \lambda$ .] In this note an affirmative answer is provided to this question.

### 1. INTRODUCTION

Let  $\mathbb{C}^{n \times m}$  denote the set of  $n \times m$  matrices over the complex field  $\mathbb{C}$ , and let  $\mathcal{H}^n$ ,  $\mathcal{H}_{\geq}^n$ , and  $\mathcal{H}_{>}^n$  denote, respectively, the set of Hermitian, nonnegative definite, and positive definite matrices of order  $n \times n$ . Given  $C \in \mathbb{C}^{n \times m}$ , let  $C^*$ ,  $C^+$ , and  $\mathcal{R}(C)$  stand for the conjugate transpose, Moore–Penrose inverse, and range, respectively, of  $C$ . For  $H, K \in \mathcal{H}^n$ , write  $H \leq K$  (or  $K \geq H$ ) if  $K - H \in \mathcal{H}_{\geq}^n$ ; this is the natural (Loewner) partial ordering of  $\mathcal{H}^n$ , induced by the closed convex cone  $\mathcal{H}_{\geq}^n$ .

It is well known that, on the set of positive definite matrices, the matrix inverse is a convex map, i.e., for every  $A, B \in \mathcal{H}_{>}^n$  and for every  $\lambda \in ]0, 1[$ , we have

$$(\lambda A + \bar{\lambda} B)^{-1} \leq \lambda A^{-1} + \bar{\lambda} B^{-1}, \quad (1.1)$$

with  $\bar{\lambda} := 1 - \lambda$ . (For the history of this matrix/operator inequality, see Nordström [7] and the references therein.)

One approach to showing inequalities of the type (1.1) is to first show it for the case  $\lambda = 1/2$ , and to then use continuity to conclude that it

---

1991 *Mathematics Subject Classification*. Primary 15A09; Secondary 26A51.

*Key words and phrases*. Generalized inverse, Jensen convexity, Loewner ordering, midpoint convexity.

Support in the form of a research grant from the Finnish Society of Sciences and Letters is gratefully acknowledged.

Download English Version:

<https://daneshyari.com/en/article/8898063>

Download Persian Version:

<https://daneshyari.com/article/8898063>

[Daneshyari.com](https://daneshyari.com)