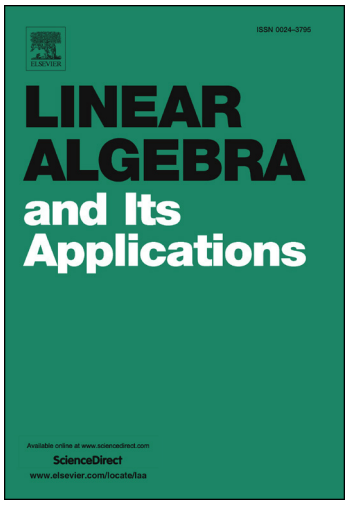
Accepted Manuscript



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PII: S0024-3795(17)30639-0
DOI: https://doi.org/10.1016/j.laa.2017.11.011
Reference: LAA 14378

To appear in: *Linear Algebra and its Applications*

Received date: 17 August 2017
Accepted date: 9 November 2017

Please cite this article in press as: S.M. Cioabă et al., Cospectral mates for the union of some classes in the Johnson association scheme, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.11.011

Cospectral mates for the union of some classes in the Johnson association scheme

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November 15, 2017

Abstract

Let $n \geq k \geq 2$ be two integers and S a subset of $\{0, 1, \ldots, k-1\}$. The graph $J_S(n, k)$ has as vertices the k-subsets of the n-set $[n] = \{1, \ldots, n\}$ and two k-subsets A and B are adjacent if $|A \cap B| \in S$. In this paper, we use Godsil-McKay switching to prove that for $m \geq 0$, $k \geq \max(m+2, 3)$ and $S = \{0, 1, ..., m\}$, the graphs $J_S(3k-2m-1, k)$ are not determined by spectrum and for $m \geq 2$, $n \geq 4m+2$ and $S = \{0, 1, ..., m\}$ the graphs $J_S(n, 2m+1)$ are not determined by spectrum. We also report some computational searches for Godsil-McKay switching sets in the union of classes in the Johnson scheme for $k \leq 5$.

1 Introduction

The spectrum of a graph G is the multi-set of eigenvalues of its adjacency matrix (see [1] for an introduction to spectral graph theory). Two graphs are called cospectral if they have the same spectrum. A graph G is determined by spectrum if any graph cospectral to G must be isomorphic to G. Two non-isomorphic graphs that are cospectral are called cospectral mates. An important research area of spectral graph theory is devoted to determining which graphs are determined by their spectra (see [4, 5] for example).

In this paper, we consider this problem for the union of classes in the Johnson association scheme. Let $n \geq k \geq 2$ be two integers and S a subset of $\{0, 1, \ldots, k-1\}$.

The graph $J_S(n, k)$ has as vertices the k-subsets of the n-set $[n] = \{1, \ldots, n\}$ and two k-subsets A and B are adjacent if $|A \cap B| \in S$.

Using this notation, the Johnson graph $J(n, k)$ is the graph $J_{\{k-1\}}(n, k)$ and the Kneser graph $K(n, k)$ is $J_{\{0\}}(n, k)$.

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