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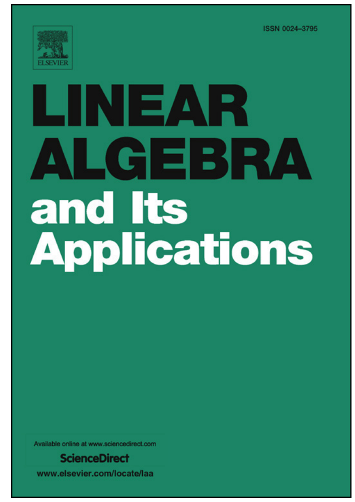

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The rank of a signed graph in terms of the rank of its underlying graph *

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Abstract

Let $\Gamma = (G, \sigma)$ be a signed graph and $A(\Gamma)$ be its adjacency matrix, where G is the underlying graph of Γ. The rank $r(\Gamma)$ of Γ is the rank of $A(\Gamma)$. We know that for a signed graph $\Gamma = (G, \sigma)$, Γ is balanced if and only if $\Gamma = (G, \sigma) \sim (G, +)$. That is, when Γ is balanced, then $r(\Gamma) = r(G)$, where $r(G)$ is the rank of the underlying graph G of Γ. A nature problem is that: how about the relations between the rank of an unbalanced signed graph and the rank of its underlying graph? In this paper, we first prove that $r(G) - 2d(G) \leq r(\Gamma) \leq r(G) + 2d(G)$ for an unbalanced signed graph with $d(G) \geq 1$, where $d(G) = |E(G)| - |V(G)| + \omega(G)$ is the dimension of cycle spaces of G, $\omega(G)$ is the number of connected components of G. As an application, we also prove that $1 - d(G) < \frac{r(\Gamma)}{r(G)} \leq 1 + d(G)$ for an unbalanced signed graph with $d(G) \geq 1$. All corresponding extremal graphs are characterized.

Key Words: Signed graphs, Rank of graphs, Dimension of cycle space.

AMS Subject Classification (2010): 05C35; 05C50.

1 Introduction

The graphs considered in this paper are finite simple graphs without multiple edges and loops. Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. The *adjacency matrix* of G of order n is defined as the $n \times n$ symmetric square matrix $A = A(G) = (a_{ij})_{n\times n}$, where $a_{ij} = 1$ if v_i is adjacent to v_j and $a_{ij} = 0$, otherwise. The *rank* $r(G)$ of G is defined to be the rank of $A(G)$, and the *nullity* $\eta(G)$ of G is defined to be the multiplicity of 0 as an eigenvalue of $A(G)$. Obviously, $n = r(G) + \eta(G)$.

A graph is called *empty* if it has some vertices and no edges. The *degree* of a vertex x in G, written as $d_G(x)$, is defined to be the number of vertices which are adjacent to x in G. Let $x \in V(G)$, x is called a *pendant* vertex if $d_G(x) = 1$ in G, and is called a *quasi-pendant* vertex if it is adjacent to a pendant vertex. An induced subgraph C_p of a graph G is called a *pendant cycle* if C_p is a cycle and has a unique vertex of degree 3 in G. Let G be a graph

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