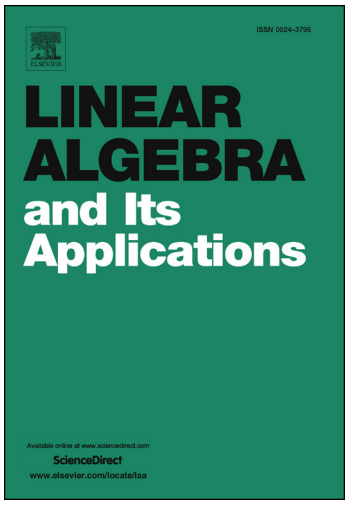

Rectangular maximum-volume submatrices and their applications

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PII: S0024-3795(17)30593-1
DOI: https://doi.org/10.1016/j.laa.2017.10.014
Reference: LAA 14355

To appear in: *Linear Algebra and its Applications*

Received date: 13 June 2016
Accepted date: 12 October 2017

Please cite this article in press as: A. Mikhalev, I.V. Oseledets, Rectangular maximum-volume submatrices and their applications, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.10.014

Rectangular maximum-volume submatrices and their applications

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Abstract

We introduce a definition of the volume of a general rectangular matrix, which is equivalent to an absolute value of the determinant for square matrices. We generalize results of square maximum-volume submatrices to the rectangular case, show a connection of the rectangular volume with an optimal experimental design and provide estimates for a growth of coefficients and an approximation error in spectral and Chebyshev norms. Three promising applications of such submatrices are presented: recommender systems, finding maximal elements in low-rank matrices and preconditioning of overdetermined linear systems. The code is available online.

Keywords: maximum volume submatrices, pseudo-skeleton approximations, CGR-approximations, recommender systems, preconditioning, optimal experimental design
2000 MSC: 15A15,41A45,65F20

1. Introduction

How to define the volume of a rectangular matrix, and how to compute a submatrix with the maximal volume in a given matrix? A standard definition of the volume of a square matrix is an absolute value of its determinant. Maximum-volume submatrices play an important role in low-rank approximations [1, 2], recommender systems [3], wireless communications [4], preconditioning of overdetermined systems [5], tensor decompositions [6]. How to compute a submatrix of exactly maximal volume is a NP-hard problem [7]. However, in many applications, a submatrix of a sufficiently large volume is enough, and it

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