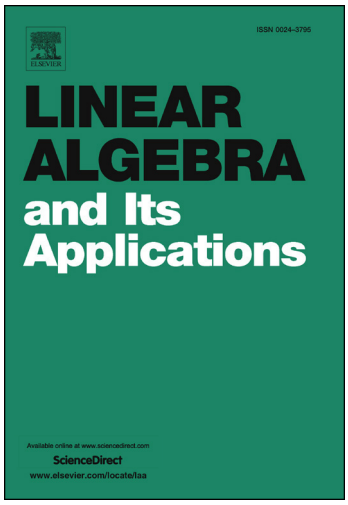

Continuous-time block-monotone Markov chains and their block-augmented truncations

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Continuous-time block-monotone Markov chains and their block-augmented truncations

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Abstract

This paper considers continuous-time block-monotone Markov chains (BMMCs) and their block-augmented truncations. We first introduce the block monotonicity and block-wise dominance relation for continuous-time Markov chains, and then provide some fundamental results on the two notions. Using these results, we show that the stationary distribution vectors obtained by the block-augmented truncation converge to the stationary distribution vector of the original BMMC. We also show that the last-column-block-augmented truncation (LC-block-augmented truncation) provides the best approximation to the stationary distribution vector of a BMMC among all the block-augmented truncations. Furthermore, we present computable upper bounds for the total variation distance between the stationary distribution vectors of a Markov chain and its LC-block-augmented truncation, under the assumption that the original Markov chain itself may not be block-monotone but is block-wise dominated by a BMMC with exponential ergodicity. Finally, we apply the obtained bounds to a queue with a batch Markovian arrival process (BMAP) and state-dependent departure rates.

Keywords: Block-monotone Markov chain (BMMC), Block-augmented truncation, Total-variation-distance error bound, GI/G/1-type Markov chain, Level-dependent quasi-birth-and-death process (LD-QBD), Exponential ergodicity
2000 MSC: 60J27, 60J22

1. Introduction

This paper considers continuous-time block-structured Markov chains characterized by an infinite number of block matrices, such as GI/G/1-type Markov