Linear Algebra and its Applications 514 (2017) 222–233

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Invariance of total nonnegativity of a matrix under entry-wise perturbation and subdirect sum of totally nonnegative matrices

Mohammad Adm [a,b], Jürgen Garloff [c,d,*]

[a] *Department of Applied Mathematics and Physics, Palestine Polytechnic University, Hebron, Palestine*
[b] *Zukunftskolleg/Department of Mathematics and Statistics, University of Konstanz, D-78464 Konstanz, Germany*
[c] *Department of Mathematics and Statistics, University of Konstanz, D-78464 Konstanz, Germany*
[d] *Institute for Applied Research, University of Applied Sciences/HTWG Konstanz, D-78405 Konstanz, Germany*

A R T I C L E   I N F O

Article history:
Received 24 May 2016
Accepted 1 November 2016
Available online 11 November 2016
Submitted by S. Fallat

MSC:
15B48

Keywords:
Totally nonnegative matrix
Entry-wise perturbation
k-subdirect sum

A B S T R A C T

A real matrix is called totally nonnegative if all of its minors are nonnegative. In this paper, the minors are determined from which the maximum allowable entry perturbation of a totally nonnegative matrix can be found, such that the perturbed matrix remains totally nonnegative. Also, the total nonnegativity of the first and second subdirect sum of two totally nonnegative matrices is considered.

* Corresponding author.
E-mail addresses: mjamathe@yahoo.com, moh_95@ppu.edu (M. Adm), garloff@htwg-konstanz.de, Juergen.Garloff@htwg-konstanz.de (J. Garloff).

http://dx.doi.org/10.1016/j.laa.2016.11.001

1. Introduction

A real matrix is called totally nonnegative (positive) if all of its minors are nonnegative (positive). For properties of these matrices the reader is referred to the two monographs [8,14] and the survey paper [6]. In the first part of our paper (taken from the dissertation [1] of the first author) we consider the question of how much the entries of a totally nonnegative matrix can be perturbed without losing the property of total nonnegativity. This question is solved in [3] for tridiagonal totally nonnegative matrices, in [5] for totally positive matrices, and in [9], see also [8, Section 9.5], for a few specified entries. A related question is whether all matrices lying between two totally nonnegative matrices, with respect to a suitable partial ordering, are totally nonnegative as well. The second author conjectured in 1982 [10] that this is true for the nonsingular totally nonnegative matrices and the so-called checkerboard ordering, see also [8, Section 3.2] and [14, Section 3.2]. In [2] we applied the so-called Cauchon algorithm to settle this conjecture.

In the second part of our paper we use the so-called condensed form of the Cauchon algorithm, see [1,4], to study special cases of the k-subdirect sum of totally nonnegative matrices [7]. We first give a short proof for the fact that the 1-subdirect sum of totally nonnegative matrices is in turn totally nonnegative [7]. The 2-subdirect sum of totally nonnegative matrices is studied in [12]. We present a counterexample which shows that a result in [12] does not hold. Finally we derive a necessary and sufficient condition for two totally nonnegative matrices given as in [12] that their 2-subdirect sum is totally nonnegative.

The organization of our paper is as follows. In the next section we explain our notation and collect some auxiliary results. In Section 3 we present our main results on the perturbation of the entries of totally nonnegative matrices. In Section 4 we give our results on the 2-subdirect sum of totally nonnegative matrices.

2. Notation and preliminary results

The set of n-by-m real matrices will be denoted by $\mathbb{R}^{n,m}$. For κ, n we denote by $Q_{\kappa,n}$ the set of all strictly increasing sequences of κ integers chosen from $\{1, 2, \ldots, n\}$. For $\alpha \in Q_{\kappa,n}$ we define $\alpha^c := \{1, \ldots, n\} \setminus \alpha$, where $\alpha^c \in Q_{n-\kappa,n}$.

For $A \in \mathbb{R}^{n,m}$, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_\kappa) \in Q_{\kappa,n}$, and $\beta = (\beta_1, \beta_2, \ldots, \beta_\mu) \in Q_{\mu,m}$, we denote by $A[\alpha|\beta]$ the κ-by-μ submatrix of A lying in the rows indexed by $\alpha_1, \alpha_2, \ldots, \alpha_\kappa$ and columns indexed by $\beta_1, \beta_2, \ldots, \beta_\mu$. We suppress the brackets when we enumerate the indices explicitly. By $A(\alpha|\beta)$ we denote the $(n-\kappa)$-by-$(m-\mu)$ submatrix $A[\alpha^c|\beta^c]$ of A. When $\alpha = \beta$, the *principal submatrix* $A[\alpha|\alpha]$ is abbreviated to $A[\alpha]$ and $\det A[\alpha]$ is called a *principal minor*, with the similar notation $A(\alpha)$ for the complementary principal submatrix.

A matrix $A \in \mathbb{R}^{n,m}$ is called *totally nonnegative* (*totally positive*) (abbreviated TN (TP)) if $\det A[\alpha|\beta] \geq 0$ (> 0), for all $\alpha \in Q_{\kappa,n}$, $\beta \in Q_{\kappa,m}$, $\kappa = 1, \ldots, n' := \min\{n, m\}$. In passing, we note that if A is TN then so are its transpose and $A^{\#} := T_n A T_n$, where