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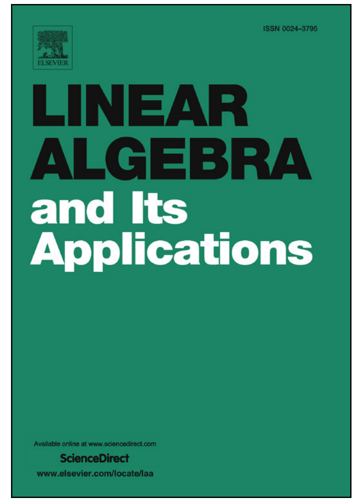

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Equitable Decompositions of Graphs with Symmetries☆

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Abstract

We investigate connections between the symmetries (automorphisms) of a graph and its spectral properties. Whenever a graph has a symmetry, i.e. a nontrivial automorphism ϕ, it is possible to use ϕ to decompose any matrix $M \in \mathbb{C}^{n\times n}$ appropriately associated with the graph. The result of this decomposition is a number of strictly smaller matrices whose collective eigenvalues are the same as the eigenvalues of the original matrix M. Some of the matrices that can be decomposed are the graph's adjaceny matrix, Laplacian matrix, etc. Because this decomposition has connections to the theory of equitable partitions it is referred to as an *equitable decomposition*. Since the graph structure of many real-world networks is quite large and has a high degree of symmetry, we discuss how equitable decompositions can be used to effectively bound both the network's spectral radius and spectral gap, which are associated with dynamic processes on the network. Moreover, we show that the techniques used to equitably decompose a graph can be used to bound the number of simple eigenvalues of undirected graphs, where we obtain sharp results of Petersdorf-Sachs type.

Keywords: Equitable Partition, Automorphism, Eigenvalue Multiplicity, Graph Symmetry
AMS Classification: 05C50

1. Introduction

In spectral graph theory one studies the relationship between two kinds of objects, a graph G (which for us may be directed or undirected) and an associated matrix M. The major aims of spectral graph theory are to determine information about a graph by examining the eigenvalues of M and inferring information about the eigenvalues from the graph structure.

The particular type of structure we consider in this paper is the notion of graph symmetries, which can be understood via graph automorphisms. Formally, a *graph automorphism* of G is defined to be a permutation $\phi : V(G) \to V(G)$ of the graph's vertices $V(G)$ that preserves adjacencies. More intuitively, a graph automorphism describes how parts of a graph can be permuted in a way that preserves the graph's overall structure. In this sense these *parts*, i.e., subgraphs, are symmetrical and together constitute a graph symmetry. An appealing result of this type is the following about eigenvalue multiplicity ([CRS09], p. 82).

Theorem 1.1 (Moshowitz; Petersdorf and Sachs)**.** *If G is a simple graph with an automorphism of order greater than 2, then the adjacency matrix for G has a multiple eigenvalue.*

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