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Topological attractors of contracting Lorenz maps ☆

Paulo Brandão

IMPA, Estrada Dona Castorina, 110, Rio de Janeiro, Brazil

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Abstract

We study the non-wandering set of contracting Lorenz maps. We show that if such a map f doesn't have any attracting periodic orbit, then there is a unique topological attractor. Furthermore, we classify the possible kinds of attractors that may occur.
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Keywords: Dynamical systems; Topological attractors; Maps of the interval; Contracting Lorenz maps

1. Introduction

In [10] Lorenz studied the solution of the system of differential equations (1) in $\mathbb{R}^3$, originated by truncating the Navier–Stokes equations for modeling atmospheric conditions

$$\begin{aligned} \dot{x} &= -10x + 10y \\ \dot{y} &= 28x - y - xz \\ \dot{z} &= -\frac{8}{3}z + xy \end{aligned} \tag{1}$$

He observed what was thought to be an attractor with features that led to the present concept of a strange attractor. V.S. Afraimovich, V.V. Bykov, L.P. Shil'nikov, in [2], and Guckenheimer and Williams, in [7], introduced the idea of Lorenz-like attractors: dynamically similar models that also displayed the characteristics of the Lorenz strange attractor.

These models consist of a hyperbolic singularity with one-dimensional unstable manifold such that, in a linearizable neighborhood, these separatrices can be considered as one of the coordinate axes, say x, in such a way that both components of $x \setminus \{0\}$ return to this neighborhood cutting transversally the plane $z = constant$, with the eigenvalues $\lambda_2 < \lambda_3 < 0 < \lambda_1$ (see Fig. 1), and the expanding condition $\lambda_3 + \lambda_1 > 0$. We consider the Poincaré map of the square $Q = \{|x| \le cte; |y| \le cte; z = cte\}$ into itself, having the returns as indicated in Fig. 1 and we can exhibit in Q a foliation by one dimensional leaves, invariant by the Poincaré map, and such that it exponentially contracts the leaves. In

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E-mail address: paulo@impa.br.

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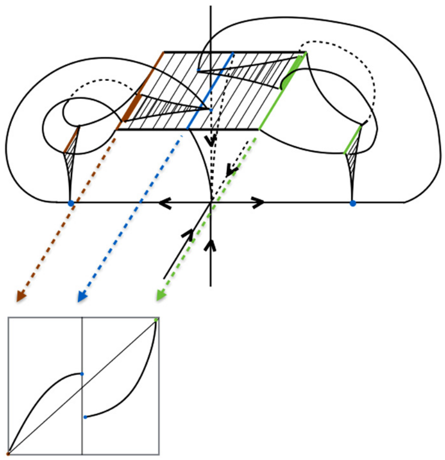

Fig. 1. Lorenz-like flow and its associated one-dimensional dynamics.

[7] Guckenheimer and Williams show that given such a system, in a neighborhood U the system is structurally stable in $codim\,2$, and in any representative family there is only a single attractor attracting the neighborhood constructed.

In [1], Arneodo, Coullet and Tresser studied similar systems, just modifying the relation between the eigenvalues of the singularity, taking $\lambda_3 + \lambda_1 < 0$: the so-called *contracting* Lorenz attractors. In this case the induced one-dimensional map is as displayed in Fig. 1.

Critical points and critical values play fundamental roles in the study of dynamics of maps of the interval and from this point of view Lorenz maps are of hybrid type. Indeed, these maps have a single critical point, as unimodal maps do, but two critical values, as bimodal ones have. Because of this, it could perhaps happen that two different attractors would occur, but indeed we prove in Theorem D that there is only one single topological attractor. That is, the behavior of contracting Lorenz maps looks like the one of unimodal maps, instead of the behavior of bimodal maps, that admits up to two attractors.

More specifically, we prove that, for contracting Lorenz maps, the possible long-term behavior scenarios for orbits of generic points are either periodic orbits, that only can be one or two of them, or a single attractor that can be one of the following types: cycle of intervals that forms a single chaotic attractor, Cherry attractor, Solenoid, or yet a subset of a chaotic Cantor set coexisting with wandering intervals. This last possibility, however, is expected not to occur, as conjectured by Martens and de Melo.

The problem addressed in this work is a conjecture of Palis in the context of contracting Lorenz maps, and the author wants to thank Jacob Palis for purposing this problem as his PhD advisor at IMPA, where this work was done, and to acknowledge all the mathematical enthusiasm shared there. The author also wants to thank V. Pinheiro, W. de Melo and C. Tresser for several fruitful conversations during the preparation of this paper.

2. Statement of the main results

We say an open interval I is *of trivial dynamics* (up to some iterate) if $\exists n \in \mathbb{N}$ such that $f^n|_I \equiv \mathrm{id}$.

Definition 2.1 *(Lorenz maps).* We say that a C^2 map $f : [0,1] \setminus \{c\} \to [0,1]$, $0 < c < 1$, is a *Lorenz map* if $f(0) = 0$, $f(1) = 1$, $f'(x) > 0, \forall x \in [0,1] \setminus \{c\}$. A Lorenz map is called *contracting* if $\lim_{x \to c} f'(x) = 0$ and there is no interval of trivial dynamics.

Given $n \geq 1$, define $f^n(c_\pm) = \lim_{0<\epsilon\to 0} f^n(c \pm \epsilon)$. The critical values of f are $f(c_-)$ and $f(c_+)$. If $x \in \{f(c_-), f(c_+)\}$ set $f^{-1}(x) = \{c\} \cup \{y \in [0,1];\ f(y) = x\}$. Given a set $X \subset [0,1]$, define $f^{-1}(X) = \bigcup_{x \in X} f^{-1}(x)$. Inductively, define $f^{-n}(x) = f^{-1}(f^{-(n-1)}(x))$, where $n \geq 2$. The *pre-orbit* of a point $x \in [0,1]$ is the set $\mathcal{O}_f^-(x) := \bigcup_{n \geq 0} f^{-n}(x)$, where $f^0(x) := x$. Denote the positive orbit of a point $x \in [0,1] \setminus \mathcal{O}_f^-(c)$ by $\mathcal{O}_f^+(x)$, i.e., $\mathcal{O}_f^+(x) = \{f^j(x);\ j \geq 0\}$. If $\exists p \geq 1$ such that $f^p(c_-) = c$, we take $p \in \mathbb{N}$ as being minimal with this property and define $\mathcal{O}_f^+(c_-) = \{f^j(c_-);\ 1 \leq j \leq p\}$. Otherwise, if $\nexists p \geq 1$ such that $f^p(c_-) = c$, we define $\mathcal{O}_f^+(c_-) = \{f^j(c_-);\ j \geq 0\}$.

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