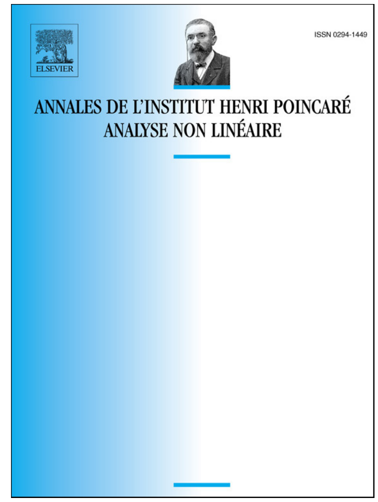

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Abstract

From minimal surfaces such as Simons' cone and catenoids, using refined Lyapunov-Schmidt reduction method, we construct new solutions for a free boundary problem whose free boundary has two components. In dimension 8, using variational arguments, we also obtain solutions which are global minimizers of the corresponding energy functional. This shows that Savin's theorem [43] is optimal.

1 Introduction

In this paper, we are interested in the following free boundary problem:

$$\begin{cases} \Delta u = 0 \text{ in } \Omega := \{-1 < u < 1\}, \\ |\nabla u| = 1 \text{ on } \partial\Omega. \end{cases} \tag{1}$$

Here the domain $\Omega \subset \mathbb{R}^n$ is a priori unspecified and $\partial\Omega$ is the free boundary. Solutions of (1) arise formally as critical points of the energy functional:

$$J(u) := \int \left[|\nabla u|^2 + \chi_{(-1,1)}(u) \right]. \tag{2}$$

In this variational formulation, the boundary condition $|\nabla u| = 1$ should be understood in some weak sense if the free boundary $\partial\Omega$ is not regular enough.