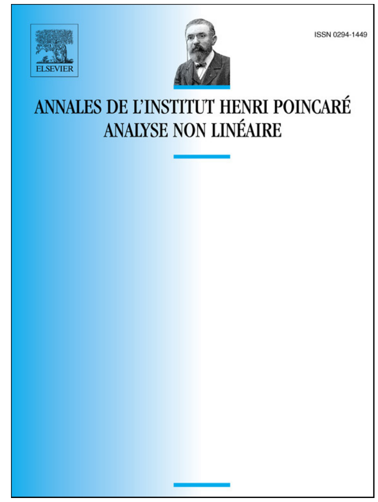
Accepted Manuscript



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PII: S0294-1449(17)30092-6
DOI: http://dx.doi.org/10.1016/j.anihpc.2017.07.002
Reference: ANIHPC 2854

To appear in: *Annales de l'Institut Henri Poincaré - Analyse non linéaire*

Received date: 10 September 2015
Revised date: 23 May 2017
Accepted date: 17 July 2017

Please cite this article in press as: K. Fellner et al., Well-posedness and exponential equilibration of a volume-surface reaction-diffusion system with nonlinear boundary coupling, *Ann. I. H. Poincaré – AN* (2017), http://dx.doi.org/10.1016/j.anihpc.2017.07.002

Well-posedness and exponential equilibration of a volume-surface reaction-diffusion system with nonlinear boundary coupling

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Abstract

We consider a model system consisting of two reaction-diffusion equations, where one species diffuses in a volume while the other species diffuses on the surface which surrounds the volume. The two equations are coupled via a nonlinear reversible Robin-type boundary condition for the volume species and a matching reversible source term for the boundary species. As a consequence of the coupling, the total mass of the two species is conserved. The considered system is motivated for instance by models for asymmetric stem cell division.

Firstly we prove the existence of a unique weak solution via an iterative method of converging upper and lower solutions to overcome the difficulties of the nonlinear boundary terms. Secondly, our main result shows explicit exponential convergence to equilibrium via an entropy method after deriving a suitable entropy entropy-dissipation estimate for the considered nonlinear volume-surface reaction-diffusion system.

Keywords: volume-surface reaction-diffusion, nonlinear boundary conditions, global existence, exponential convergence to equilibrium
2010 MSC: 35K61, 35A01, 35B40, 35K57

1. Introduction

In this paper, we consider a nonlinear volume-surface reaction-diffusion system, which couples a non-negative volume-concentration $u(x,t)$ diffusing on a bounded domain $\Omega \subset \mathbb{R}^N (N \geq 1)$ with a non-negative surface-concentration $v(x,t)$ diffusing on the sufficiently smooth boundary $\Gamma := \partial\Omega$ of Ω (e.g. $\partial\Omega \in C^{2+\epsilon}$ for $\epsilon > 0$).

The interface conditions connecting these two concentrations are a nonlinear Robin-type boundary condition for the volume-concentration $u(x,t)$ and a matching reversible reaction source term in the equation for the surface-concentration $v(x,t)$:

$$\begin{cases} u_t - \delta_u \Delta u = 0, & x \in \Omega, t > 0, \\ \delta_u \frac{\partial u}{\partial \nu} = -\alpha(k_u u^\alpha - k_v v^\beta), & x \in \Gamma, t > 0, \\ v_t - \delta_v \Delta_\Gamma v = \beta(k_u u^\alpha - k_v v^\beta), & x \in \Gamma, t > 0, \\ u(0,x) = u_0(x) \geq 0, & x \in \Omega, \\ v(0,x) = v_0(x) \geq 0, & x \in \Gamma. \end{cases} \tag{1.1}$$

Here, we denote by Δ the Laplace operator on Ω with a positive diffusion coefficient $\delta_u > 0$ and by Δ_Γ the Laplace-Beltrami operator on Γ (see e.g. [1]) with a non-negative diffusion coefficient $\delta_v \geq 0$, and $\nu(x)$ denotes the unit outward normal vector of Γ at the point x. Moreover, we shall consider nonnegative initial concentrations $u_0(x) \geq 0$ on Ω and $v_0(x) \geq 0$ on Γ.

The stoichiometric coefficients $\alpha, \beta \in [1, \infty)$ together with the positive, bounded reaction rates $k_u(t,x)$, $k_v(t,x) \in L^\infty_+([0,\infty) \times \Gamma)$ characterise the key feature of the model system (1.1), which is the nonlinear

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