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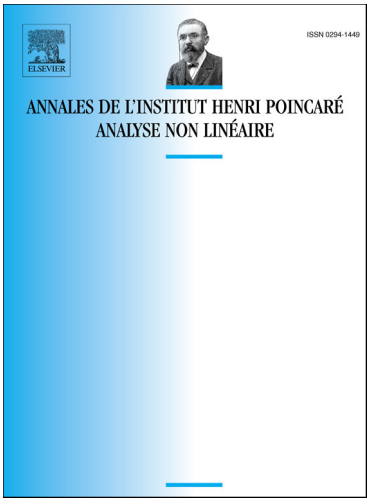

Weak solutions of semilinear elliptic equation involving Dirac mass

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WEAK SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATION INVOLVING DIRAC MASS

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Abstract. In this paper, we study the elliptic problem with Dirac mass

$$\begin{cases} -\Delta u = Vu^p + k\delta_0 \quad \text{in} \quad \mathbb{R}^N, \\ \lim\limits_{|x|\to+\infty} u(x) = 0, \end{cases} \tag{1}$$

where $N > 2$, $p > 0$, $k > 0$, δ_0 is the Dirac mass at the origin and the potential V is locally Lipchitz continuous in $\mathbb{R}^N \setminus \{0\}$, with non-empty support and satisfying

$$0 \le V(x) \le \frac{\sigma_1}{|x|^{a_0}(1+|x|^{a_\infty - a_0})},$$

with $a_0 < N$, $a_0 < a_\infty$ and $\sigma_1 > 0$. We obtain two positive solutions of (1) with additional conditions for parameters on a_∞, a_0, p and k. The first solution is a minimal positive solution and the second solution is constructed via Mountain Pass Theorem.

1. Introduction

The goal of this paper is to study the existence of multiple weak solutions to the nonlinear elliptic problem with Dirac mass

$$(P_k) \qquad \begin{cases} -\Delta u = Vu^p + k\delta_0 \quad \text{in } \mathbb{R}^N, \\ \lim\limits_{|x|\to+\infty} u(x) = 0 \end{cases}$$

where $N > 2$, $p > 0$, $k > 0$, δ_0 is the Dirac mass at the origin, and the potential V is locally Lipchitz continuous in $\mathbb{R}^N \setminus \{0\}$. Problem (P_k) concerns with source term in contrast with the problem with absorption, namely the semi-linear elliptic equation

$$\begin{cases} -\Delta u + g(u) = \nu \quad \text{in} \quad \Omega, \\ \qquad\qquad u = 0 \quad \text{on} \quad \partial\Omega, \end{cases} \tag{1.1}$$

AMS Subject Classifications: 35J60, 35J20.
Key words: Weak solution, Mountain Pass theorem, Dirac mass.