

# Accepted Manuscript

Boundary observability of gravity water waves

Thomas Alazard

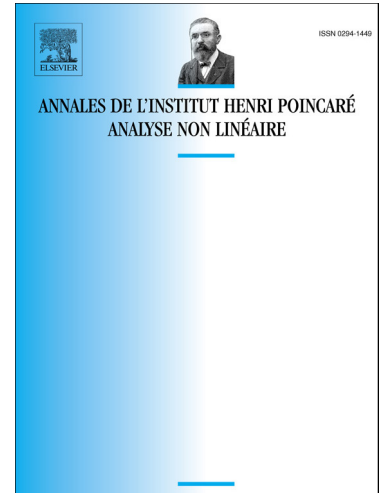
PII: S0294-1449(17)30097-5  
DOI: <http://dx.doi.org/10.1016/j.anihpc.2017.07.006>  
Reference: ANIHPC 2859

To appear in: *Annales de l'Institut Henri Poincaré - Analyse non linéaire*

Received date: 2 July 2015  
Revised date: 19 July 2017  
Accepted date: 20 July 2017

Please cite this article in press as: T. Alazard, Boundary observability of gravity water waves, *Ann. I. H. Poincaré – AN* (2017), <http://dx.doi.org/10.1016/j.anihpc.2017.07.006>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Boundary observability of gravity water waves

Thomas Alazard  
CNRS & École normale supérieure

## Abstract

Consider a three-dimensional fluid in a rectangular tank, bounded by a flat bottom, vertical walls and a free surface evolving under the influence of gravity. We prove that one can estimate its energy by looking only at the motion of the points of contact between the free surface and the vertical walls. The proof relies on the multiplier technique, the Craig-Sulem-Zakharov formulation of the water-wave problem, a Pohozaev identity for the Dirichlet to Neumann operator, previous results about the Cauchy problem and computations inspired by the analysis done by Benjamin and Olver of the conservation laws for water waves.

## 1 Introduction

Consider surface waves over an incompressible liquid, evolving under the influence of gravity, in the case where the fluid is located inside a fixed rectangular tank  $\mathcal{R}$  of the form  $\mathcal{R} = Q \times [-h, +\infty)$  where  $Q = [0, L_1] \times [0, L_2]$  and  $h$  is a positive constant. At time  $t$ , the fluid domain  $\Omega(t)$  is given by

$$\Omega(t) = \{ (x, y) : x \in Q, -h \leq y \leq \eta(t, x) \}, \quad (1)$$

where  $x = (x_1, x_2)$  (resp.  $y$ ) is the horizontal (resp. vertical) space variable. The equations which dictate the motion are the incompressible Euler equations with free surface. This is a system of two nonlinear equations: the incompressible Euler equation for the velocity potential  $\phi: \Omega(t) \rightarrow \mathbb{R}$  (so that the velocity is  $v = \nabla_{x,y}\phi$ ) and a kinematic equation for  $\eta$  which states that the free surface moves with the fluid. The energy, which is the sum of the potential energy and the kinetic energy, is conserved:

$$\frac{d\mathcal{H}}{dt} = 0 \quad \text{with} \quad \mathcal{H} = \frac{g}{2} \int_Q \eta^2(t, x) dx + \frac{1}{2} \iint_{\Omega(t)} |\nabla_{x,y}\phi(t, x, y)|^2 dx dy, \quad (2)$$

where  $g$  is the acceleration of gravity. This paper is devoted to the analysis of the following question: is it possible to estimate the energy  $\mathcal{H}$  of gravity water waves by looking only at the motion of some of the curves of contact between the free surface and the vertical walls? From the point of view of control theory, this is the question of boundary observability of gravity water waves.

---

This work is partly supported by the grant “ANAÉ” ANR-13-BS01-0010-03.

Download English Version:

<https://daneshyari.com/en/article/8898134>

Download Persian Version:

<https://daneshyari.com/article/8898134>

[Daneshyari.com](https://daneshyari.com)