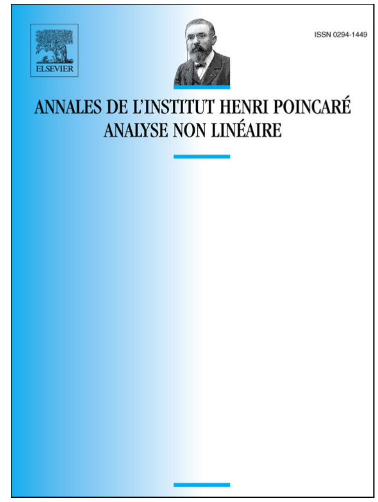

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Abstract

Consider a three-dimensional fluid in a rectangular tank, bounded by a flat bottom, vertical walls and a free surface evolving under the influence of gravity. We prove that one can estimate its energy by looking only at the motion of the points of contact between the free surface and the vertical walls. The proof relies on the multiplier technique, the Craig-Sulem-Zakharov formulation of the water-wave problem, a Pohozaev identity for the Dirichlet to Neumann operator, previous results about the Cauchy problem and computations inspired by the analysis done by Benjamin and Olver of the conservation laws for water waves.

1 Introduction

Consider surface waves over an incompressible liquid, evolving under the influence of gravity, in the case where the fluid is located inside a fixed rectangular tank $\mathscr{R}$ of the form $\mathscr{R} = Q \times [-h, +\infty)$ where $Q = [0, L_1] \times [0, L_2]$ and h is a positive constant. At time t, the fluid domain $\Omega(t)$ is given by

$$\Omega(t) = \{ (x, y) \, : \, x \in Q, \ -h \le y \le \eta(t, x) \}, \tag{1}$$

where $x = (x_1, x_2)$ (resp. y) is the horizontal (resp. vertical) space variable. The equations which dictate the motion are the incompressible Euler equations with free surface. This is a system of two nonlinear equations: the incompressible Euler equation for the velocity potential $\phi \colon \Omega(t) \to \mathbb{R}$ (so that the velocity is $v = \nabla_{x,y}\phi$) and a kinematic equation for η which states that the free surface moves with the fluid. The energy, which is the sum of the potential energy and the kinetic energy, is conserved:

$$\frac{d\mathcal{H}}{dt} = 0 \quad \text{with} \quad \mathcal{H} = \frac{g}{2} \int_Q \eta^2(t, x) \, dx + \frac{1}{2} \iint_{\Omega(t)} \left| \nabla_{x,y}\phi(t, x, y) \right|^2 \, dx dy, \tag{2}$$

where g is the acceleration of gravity. This paper is devoted to the analysis of the following question: is it possible to estimate the energy $\mathcal{H}$ of gravity water waves by looking only at the motion of some of the curves of contact between the free surface and the vertical walls? From the point of view of control theory, this is the question of boundary observability of gravity water waves.

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