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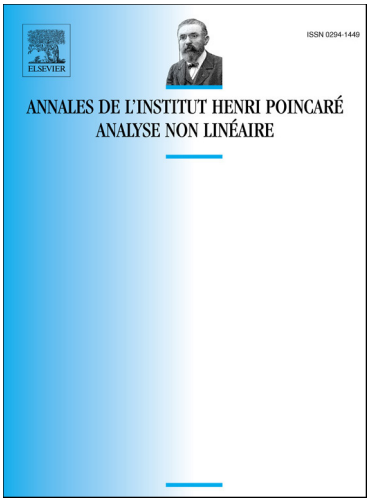


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# Existence of a minimal non-scattering solution to the mass-subcritical generalized Korteweg-de Vries equation

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**Abstract**

In this article, we prove the existence of a non-scattering solution, which is minimal in some sense, to the mass-subcritical generalized Korteweg-de Vries (gKdV) equation in the scale critical $\hat{L}^r$ space where $\hat{L}^r = \{f \in \mathcal{S}'(\mathbb{R}) | \, \|f\|_{\hat{L}^r} = \|\hat{f}\|_{L^{r'}} < \infty\}$. We construct this solution by a concentration compactness argument. Then, key ingredients are a linear profile decomposition result adopted to $\hat{L}^r$-framework and approximation of solutions to the gKdV equation which involves rapid linear oscillation by means of solutions to the nonlinear Schrödinger equation.

*Keywords:* generalized Korteweg-de Vries equation, scattering problem, threshold solution
*2010 MSC:* Primary 35Q53, 35B40; Secondary 35B30

## 1. Introduction

In this article, we consider the generalized Korteweg-de Vries (gKdV) equation

$$\begin{cases} \partial_t u + \partial_x^3 u = \mu \partial_x (|u|^{2\alpha} u), & t, x \in \mathbb{R}, \\ u(0,x) = u_0(x) \in \hat{L}^\alpha(\mathbb{R}), & x \in \mathbb{R} \end{cases} \tag{gKdV}$$

where $u : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is an unknown function, $u_0 : \mathbb{R} \to \mathbb{R}$ is a given data, and $\mu = \pm 1$ and $\alpha > 0$ are constants. The space $\hat{L}^r$ is defined for $1 \leqslant r \leqslant \infty$ by

$$\hat{L}^r = \hat{L}^r(\mathbb{R}) := \{f \in \mathcal{S}'(\mathbb{R}) | \, \|f\|_{\hat{L}^r} = \|\hat{f}\|_{L^{r'}} < \infty\},$$

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