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Analysis of miscible displacement through porous media with vanishing molecular diffusion and singular wells ☆

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Abstract

This article proves the existence of solutions to a model of incompressible miscible displacement through a porous medium, with zero molecular diffusion and modelling wells by spatial measures. We obtain the solution by passing to the limit on problems indexed by vanishing molecular diffusion coefficients. The proof employs cutoff functions to excise the supports of the measures and the discontinuities in the permeability tensor, thus enabling compensated compactness arguments used by Y. Amirat and A. Ziani for the analysis of the problem with L^2 wells (Amirat and Ziani, 2004 [1]). We give a novel treatment of the diffusion–dispersion term, which requires delicate use of the Aubin–Simon lemma to ensure the strong convergence of the pressure gradient, owing to the troublesome lower-order terms introduced by the localisation procedure.

Keywords: Flow in porous medium; Elliptic–parabolic system; Degenerate equations; Existence; Measure data; Vanishing diffusion

1. Introduction

1.1. The miscible displacement problem

We study the single-phase, miscible displacement of one incompressible fluid by another through a porous medium, as occurs in enhanced oil recovery processes. Neglecting gravity, the model reads [10,18]

$$\left.\begin{aligned} \mathbf{u}(x,t) &= -\frac{\mathbf{K}(x)}{\mu(c(x,t))}\nabla p(x,t) \\ \operatorname{div}\mathbf{u}(x,t) &= (q^I - q^P)(x,t) \end{aligned}\right\}, \quad (x,t)\in\Omega\times(0,T), \tag{1.1a}$$

$$\Phi(x)\partial_t c(x,t) - \operatorname{div}\big(\mathbf{D}(x,\mathbf{u}(x,t))\nabla c - c\mathbf{u}\big)(x,t) + (q^P c)(x,t) = (q^I\hat{c})(x,t), \quad (x,t)\in\Omega\times(0,T), \tag{1.1b}$$

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subject to the no-flow boundary conditions

$$\mathbf{u}(x,t)\cdot\mathbf{n}=0,\quad (x,t)\in\partial\Omega\times(0,T),\text{ and} \tag{1.1c}$$

$$\mathbf{D}(x,\mathbf{u}(x,t))\nabla c(x,t)\cdot\mathbf{n}=0,\quad (x,t)\in\partial\Omega\times(0,T), \tag{1.1d}$$

the initial condition

$$c(x,0)=c_0(x),\qquad x\in\Omega, \tag{1.1e}$$

and a normalisation condition to eliminate arbitrary constants in the solution p of the elliptic equation (1.1a):

$$\int_\Omega p(x,t)\,\mathrm{d}x=0\quad\text{for all } t\in(0,T). \tag{1.1f}$$

The unknowns of the system are the pressure p and Darcy velocity $\mathbf{u}$ of the fluid mixture, and the concentration c of one of the components in the fluid mixture. The reservoir is represented by Ω, a bounded connected open subset of $\mathbb{R}^d$, $d=2$ or 3, and the recovery process occurs over the time interval $(0,T)$. The reservoir-dependent quantities of porosity and absolute permeability are Φ and $\mathbf{K}$, respectively. We denote by q^I and q^P the sums of injection well source terms and production well sink terms (henceforth collectively referred to as source terms), respectively, and write $\hat{c}$ for the concentration of the injected fluid.

The coefficient $\mathbf{D}$ in (1.1b) is the diffusion–dispersion tensor, derived by Peaceman [17] as

$$\mathbf{D}(x,\mathbf{u})=\Phi(x)\Big(d_m\mathbf{I}+|\mathbf{u}|\Big(d_l E(\mathbf{u})+d_t(\mathbf{I}-E(\mathbf{u}))\Big)\Big), \tag{1.1g}$$

where

$$E(\mathbf{u})=\left(\frac{\mathbf{u}_i\mathbf{u}_j}{|\mathbf{u}|^2}\right)_{1\le i,j\le d} \tag{1.1h}$$

is the projection in the direction of flow. The constants d_m, d_l and d_t are the molecular diffusion coefficient and the longitudinal and transverse mechanical dispersion coefficients, respectively. After Koval [16] (see also [5,20]), the concentration-dependent viscosity μ of the fluid mixture often assumes the form

$$\mu(c)=\mu(0)\left(1+\left(M^{1/4}-1\right)c\right)^{-4}\qquad\text{for } c\in[0,1], \tag{1.1i}$$

where the mobility ratio $M:=\frac{\mu(0)}{\mu(1)}>1$. Finally, the boundary condition (1.1c) enforces a compatibility condition upon the source terms:

$$\int_\Omega q^I(x,t)\,\mathrm{d}x=\int_\Omega q^P(x,t)\,\mathrm{d}x\quad\text{for all } t\in(0,T). \tag{1.1j}$$

1.2. Principal contributions

Our main result, Theorem 2.2, is the existence of weak solutions to (1.1) when $d_m=0$ and q^I and q^P are modelled spatially as bounded, nonnegative Radon measures on Ω. Indeed, the novelty of this article is the presence of both these features simultaneously; Amirat and Ziani [1] analyse the system as $d_m\to 0$ with $q^I, q^P\in L^\infty(0,T;L^2(\Omega))$, and our previous work [9] establishes existence for $d_m>0$ and measure source terms. Fabrie and Gallouët [11] assume that the diffusion–dispersion tensor is uniformly bounded to address the latter scenario. The first existence result for (1.1) as written above is due to Feng [12], focussing mostly on the two-dimensional problem with sources in $L^\infty(0,T;L^2(\Omega))$. The subsequent analysis of Chen and Ewing [5] is valid for very general boundary conditions in three dimensions, but assumes $d_m>0$ and regular source terms. Uniqueness is known for "strong" solutions [12], but appears to be open for weak solutions even with $d_m>0$ fixed [1,5,12].

We prove Theorem 2.2 by passing to the limit as $d_m\to 0$ on a sequence of problems with measure source terms defined in Section 3. In further contrast to Amirat–Ziani who take $\Phi\equiv 1$ and $\mathbf{K}$ continuous, we only assume that the porosity is bounded, and we allow for discontinuous permeabilities of the kind that one expects in practice [6].

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