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3-D axisymmetric subsonic flows with nonzero swirl for the compressible Euler–Poisson system

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Abstract

We address the structural stability of 3-D axisymmetric subsonic flows with nonzero swirl for the steady compressible Euler–Poisson system in a cylinder supplemented with non-small boundary data. A special Helmholtz decomposition of the velocity field is introduced for 3-D axisymmetric flow with a nonzero swirl (= angular momentum density) component. With the newly introduced decomposition, a quasilinear elliptic system of second order is derived from the elliptic modes in Euler–Poisson system for subsonic flows. Due to the nonzero swirl, the main difficulties lie in the solvability of a singular elliptic equation which concerns the angular component of the vorticity in its cylindrical representation, and in analysis of streamlines near the axis $r = 0$.

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Keywords: Steady Euler–Poisson system; Axisymmetric; Swirl; Subsonic; Helmholtz decomposition; Elliptic system; Singular elliptic equation; Transport equation

1. Introduction and main results

The steady Euler–Poisson system

$$\begin{cases} \text{div }(\rho\mathbf{u}) = 0, \\ \text{div }(\rho\mathbf{u}\otimes\mathbf{u} + pI_n) = \rho\nabla\Phi, \\ \text{div }(\rho\mathcal{E}\mathbf{u} + p\mathbf{u}) = \rho\boldsymbol{u}\cdot\nabla\Phi, \\ \Delta_{\rm x}\Phi = \rho - b(\rm x), \end{cases} \tag{1.1}$$

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is a hydrodynamical model of semiconductor devices or plasmas, describing local behaviors of the electron density ρ, the macroscopic particle velocity $\mathbf{u}$, and the total energy $\mathcal{E} = |\mathbf{u}|^2/2 + e$, where e is the internal energy. The first equation, which is also called as the continuity equation, expresses the conservation of electrons, the second equations express the conservation of momentum, where $\rho\nabla\Phi$ is the Coulomb force of electron particles. The third equation expresses the conservation of energy, and the last Poisson equation expresses the local change of the electric potential Φ due to the volumetric charge density. The function $b(\mathrm{x}) > 0$ is the prescribed density of fixed, positively charged background ions. Physically, by solving the Euler–Poisson equations in predetermined macroscopic device region with the relevant boundary conditions, we get the electric distribution or electric current in any proper cross sections.

To close the system (1.1), we introduce the equation of state

$$p = p(\rho, e) = (\gamma - 1)\rho e, \tag{1.2}$$

where $\gamma > 1$ is called the *adiabatic constant*. In terms of the entropy S, one also has

$$p(\rho, S) = A\exp\left(\frac{S}{c_v}\right)\rho^{\gamma}, \tag{1.3}$$

where A and c_v are positive constants. For more details about the physical background of the semiconductor device or models, one may refer to [25–27].

Define Bernoulli's function $\mathcal{B}$ by

$$\mathcal{B} = \frac{|\mathbf{u}|^2}{2} + e + \frac{p}{\rho} = \frac{|\mathbf{u}|^2}{2} + \frac{A\gamma}{\gamma - 1} exp\left(\frac{S}{c_v}\right)\rho^{\gamma-1}. \tag{1.4}$$

Then, the system (1.1) can be rewritten as

$$\begin{cases} \text{div } (\rho\mathbf{u}) = 0, \\ \text{div } (\rho\mathbf{u} \otimes \mathbf{u} + pI_n) = \rho\nabla\Phi, \\ \text{div } (\rho\mathbf{u}\mathcal{B}) = \rho\mathbf{u}\cdot\nabla\Phi, \\ \Delta_{\mathrm{x}}\Phi = \rho - b(\mathbf{x}). \end{cases} \tag{1.5}$$

The system (1.5) is a hyperbolic–elliptic coupled system, and behaves quite differently in subsonic states ($|\mathbf{u}| < \sqrt{\partial_\rho p(\rho, S)}$) and supersonic states ($|\mathbf{u}| > \sqrt{\partial_\rho p(\rho, S)}$), respectively. The goal of this work is to prove the structural stability of three dimensional axially symmetric subsonic flows with nonzero swirl (= nonzero angular momentum) to the system (1.5) in a circular cylinder of finite length without assumptions of small momentum or small flow speed. The existence and the uniqueness of subsonic flows to Euler–Poisson system were proved in [1,3–6,10,11,26,29,31]. In [10,11], the unique existence of subsonic flows for Euler–Poisson system is proved for small data. Subsonic flows with small current flux were studied in [1,3,26,31]. The structural stability of subsonic flows for multidimensional potential flow and two dimensional flow with nonzero vorticity was proved in [4–6], where no smallness of data was assumed. In [29], the unique existence of three dimensional subsonic flows with nonzero vorticity was proved. It used the Bernoulli's law to provide a new formulation of Euler–Poisson equations by reducing the dimension of the velocity, this idea is originally from [28]. Although the method in [29] works for the 3-D non-isentropic Euler–Poisson system, there are some smallness requirements on the background solutions.

The new feature of this work is that we construct three dimensional subsonic flows with nonzero vorticity, and that no smallness of data is required. In [4], it is found that a special structure of potential flow model of Euler–Poisson system yields the structural stability of multidimensional subsonic solutions without assumption of smallness of data. This result is extended to the case of two dimensional flow with nonzero vorticity through a two dimensional Helmholtz decomposition $\mathbf{u} = \nabla\varphi + \nabla^{\perp}\psi$ in [5]. In this paper, we introduce a Helmholtz decomposition for three dimensional subsonic flows in the form of

$$\mathbf{u} = \nabla\varphi + \text{curl}\mathbf{V} \quad \text{with } \mathbf{V} = h\mathbf{e}_r + \psi\mathbf{e}_\theta,$$

where φ, h, ψ are functions of (x, r) for $r = \sqrt{x_2^2 + x_3^2}$. With using this decomposition, we investigate axisymmetric subsonic flows with nonzero vorticity. In particular, the function ψ concerns the swirl (= angular momentum density). There are many other studies of axially symmetric smooth subsonic solutions to the steady compressible Euler

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