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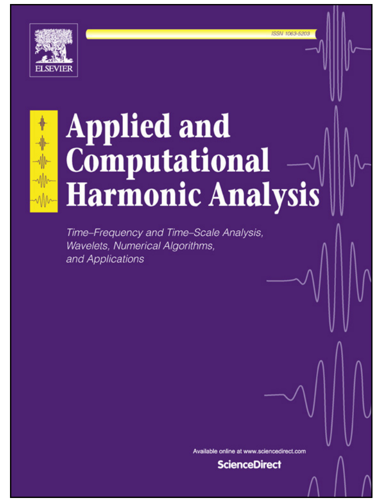

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COMPRESSED SENSING OF DATA WITH A KNOWN DISTRIBUTION

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ABSTRACT. Compressed sensing is a technique for recovering an unknown sparse signal from a small number of linear measurements. When the measurement matrix is random, the number of measurements required for perfect recovery exhibits a phase transition: there is a threshold on the number of measurements after which the probability of exact recovery quickly goes from very small to very large. In this work we are able to reduce this threshold by incorporating statistical information about the data we wish to recover. Our algorithm works by minimizing a suitably weighted ℓ_1-norm, where the weights are chosen so that the expected statistical dimension of the corresponding descent cone is minimized. We also provide new discrete-geometry-based Monte Carlo algorithms for computing intrinsic volumes of such descent cones, allowing us to bound the failure probability of our methods.

1. INTRODUCTION

The sensing problem consists on trying to recover a signal $\mathbf{x}_0 \in \mathbb{R}^d$ from m linear measurements encoded in a vector $\mathbf{y}_0 := \mathbf{A}\mathbf{x}_0$, where $\mathbf{A}$ is a given $m \times d$ matrix with $m < d$. In the seminal works by Candès, Romberg, and Tao [CT05, CRT06] and Donoho [Don06], the following convex optimization algorithm is proposed as a possible solution:

$$\text{(P)} \qquad \Delta(\mathbf{y}_0) := \arg\min_{\mathbf{x} \in \mathbb{R}^d} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{y}_0.$$

We say that the problem (P) is *successful* or that it performs a *perfect recovery* for $\mathbf{A}$ and $\mathbf{x}_0$ if it has a unique solution and this solution is $\mathbf{x}_0$. We cannot expect this method to work for arbitrary signals and measurements; by taking m strictly less than d we are collapsing dimensions and consequently losing information. However, if $\mathbf{A} \in \mathbb{R}^{m \times d}$ is a random matrix with independent Gaussian entries, it is shown in [CRT06, Don06] that this method is successful with very high probability for all sufficiently sparse vectors, i.e., vectors with a low number of non-zero entries.

These success guarantees were obtained by proving that matrices with Gaussian entries satisfy the so-called Restricted Isometry Property with high probability (for suitable choices of m and d), and that this condition is sufficient to guarantee that (P) is successful for all sufficiently sparse vectors $\mathbf{x}_0$.

Although the Restricted Isometry Property is a sufficient condition for (P) to be successful, it does not explain the phase transition phenomenon exhibited by

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