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PROMP: A sparse recovery approach to lattice-valued signals

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Applications such as wireless communications require efficient sensing techniques of signals with the a priori knowledge of those being lattice-valued. In this paper, we study the impact of this prior information on compressed sensing methodologies, and introduce and analyze PROMP ("PReprojected Orthogonal Matching Pursuit") as a novel algorithmic approach for sparse recovery of lattice-valued signals. More precisely, we first show that the straightforward approach to project the solution of Basis Pursuit onto a prespecified lattice does not improve the performance of Basis Pursuit in this situation. We then introduce PROMP as a novel sparse recovery algorithm for lattice-valued signals which has very low computational complexity, alongside a detailed mathematical analysis of its performance and stability under noise. Finally, we present numerical experiments which show that PROMP outperforms standard sparse recovery approaches in the lattice-valued signal regime.

1. Introduction

During the last 10 years, the area of compressed sensing or, more generally, sparse recovery has matured to a novel research area intersecting, in particular, mathematics, computer science, and electrical engineering. Its main objective is to efficiently solve underdetermined linear equations

$$Ax = b,$$

with $A \in \mathbb{R}^{m,d}$ and $b \in \mathbb{R}^m$, $m < d$ under the additional assumption that the solution $x \in \mathbb{R}^d$ is *sparse*. The feasibility of this hypothesis is by now generally accepted, and sparsity of data can be identified as a new paradigm in signal and image processing. The most basic notion of sparsity of a vector $x = (x_1, \ldots, x_d)$ states that x is called *s-sparse* provided that the number of non-zero elements x_i is less than or equal to s.

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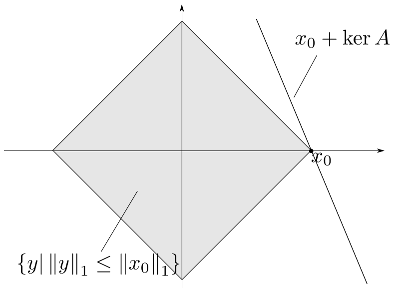

Fig. 1. Sparse signals x_0 lie on low-dimensional faces of the ℓ_1 unit ball.

In this situation, sufficient conditions – typically in terms of incoherence properties of the measurement matrix A and the sparsity of x – for precise recovery of the signal b by, for instance, convex optimization algorithms are known even when the measurements are contaminated with noise. We refer the interested reader to [11] for a survey.

In many applications however, additional information of the original signal is known and should be exploited to improve recovery guarantees. In this paper, we will focus on the situation of lattice-valued signals, which appear, for instance, in massive MIMO [37], wideband spectrum sensing [2] and error correcting codes [5]. Previous work on sparse recovery aspects of this situation has however mostly focused on integer-valued or binary signals and without a detailed mathematical analysis of recovery guarantees. Hence, a sparse recovery algorithm for lattice-valued signals alongside a precise analysis of its performance also under noise is still an open problem.

1.1. Algorithms for sparse recovery

The most standard approach to the sparse recovery problem is the Basis Pursuit algorithm, [8] or rather method, which consists of searching for the solution to the equation $Ax = b$ having the smallest ℓ_1-norm, i.e.,

$$\|x\|_1 \text{ s.t. } Ax = b. \qquad (\mathcal{P}_1)$$

A beautiful geometrical intuition stands behind this approach: Sparse vectors x_0 with unit ℓ_1-norm lie on low-dimensional faces of the cross polytope $T^d = \{x \mid \|x\|_1 = 1\}$. Thus it is probable that the set $x_0 + \ker A$ only touches T^d in x_0 (Fig. 1). By using random matrices such as Gaussian iid, it is possible to achieve a very high recovery probability given that the number of measurements satisfies $m \gtrsim s\log(d)$, where s is the sparsity of the signal x_0 [6].

The sparse recovery algorithm Orthogonal Matching Pursuit (Algorithm 1) is of different nature [36]. Its objective is to iteratively construct a support estimate S. In each step, one greedily chooses a new index i to minimize

$$\min_{\text{supp } v \subseteq S \cup i} \|Av - b\|_2 .$$

This algorithm also requires an order of $s\log(d)$ measurements to succeed at recovering an s-sparse vector x_0 [45]. Empirically, it performs slightly worse than Basis Pursuit when it comes to recovery probabilities. Main advantages of Orthogonal Matching Pursuit are though that this algorithm is very fast and easy to implement. [44]. Since this algorithm will be a backbone of the algorithmic approach being developed in our paper, we briefly state its pseudo-code version.

Certainly, a variety of other algorithm performing sparse recovery are available, and we refer to [11] for a survey. Since in the sequel Basis Pursuit and Orthogonal Matching Pursuit will play the key roles, we for now refrain from detailing other approaches.

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