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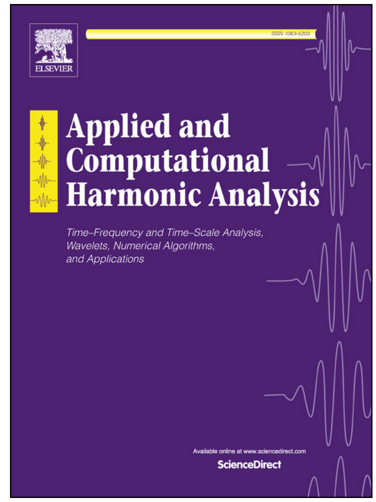

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Stability and super-resolution of generalized spike recovery

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Abstract

We consider the problem of recovering a linear combination of Dirac delta functions and derivatives from a finite number of Fourier samples corrupted by noise. This is a generalized version of the well-known spike recovery problem, which is receiving much attention recently. We analyze the numerical conditioning of this problem in two different settings depending on the order of magnitude of the quantity $N\eta$, where N is the number of Fourier samples and η is the minimal distance between the generalized spikes. In the "well-conditioned" regime $N\eta \gg 1$, we provide upper bounds for first-order perturbation of the solution to the corresponding least-squares problem. In the near-colliding, or "super-resolution" regime $N\eta \to 0$ with a single cluster, we propose a natural regularization scheme based on decimating the samples – essentially increasing the separation η – and demonstrate the effectiveness and near-optimality of this scheme in practice.

Keywords: Spike recovery, Prony system, super-resolution, decimation, numerical conditioning

1. Introduction

In this work we consider the problem of reconstructing the locations $\xi_j \in [-\pi, \pi]$ and amplitudes $c_{\ell,j} \in \mathbb{R}$ of a "generalized spike train"

$$f(x) = \sum_{j=1}^{\mathcal{K}} \sum_{\ell=0}^{\ell_j - 1} c_{\ell,j} \delta^{(\ell)}(x - \xi_j), \tag{1}$$

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