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A new hybrid integral representation for frequency domain scattering in layered media

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ABSTRACT

A variety of problems in acoustic and electromagnetic scattering require the evaluation of impedance or layered media Green's functions. Given a point source located in an unbounded half-space or an infinitely extended layer, Sommerfeld and others showed that Fourier analysis combined with contour integration provides a systematic and broadly effective approach, leading to what is generally referred to as the Sommerfeld integral representation. When either the source or target is at some distance from an infinite boundary, the number of degrees of freedom needed to resolve the scattering response is very modest. When both are near an interface, however, the Sommerfeld integral involves a very large range of integration and its direct application becomes unwieldy. Historically, three schemes have been employed to overcome this difficulty: the method of images, contour deformation, and asymptotic methods of various kinds. None of these methods make use of classical layer potentials in physical space, despite their advantages in terms of adaptive resolution and high-order accuracy. The reason for this is simple: layer potentials are impractical in layered media or half-space geometries since they require the discretization of an infinite boundary. In this paper, we propose a hybrid method which combines layer potentials (physical-space) on a finite portion of the interface together with a Sommerfeld-type (Fourier) correction. We prove that our method is efficient and rapidly convergent for arbitrarily located sources and targets, and show that the scheme is particularly effective when solving scattering problems for objects which are close to the half-space boundary or even embedded across a layered media interface.

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1. Introduction

Acoustic and electromagnetic wave scattering problems in half-space or layered media geometries require the solution of the governing partial differential equations subject to suitable boundary and radiation conditions. In the two-dimensional, time-harmonic setting, both reduce to the Helmholtz equation (Fig. 1)

$$\Delta u + k^2 u = f, \tag{1.1}$$

with boundary conditions enforced on a scatterer Ω_0 and interface conditions enforced on the line $y = 0$, either of impedance (Robin) type

$$\frac{\partial u}{\partial n} + ik\alpha u = 0, \tag{1.2}$$

or of transmission type

$$[u] = 0, \qquad \left[\frac{\partial u}{\partial n}\right] = 0. \tag{1.3}$$

The Helmholtz coefficient k is given as $k = \omega/c$, where ω is the governing frequency and c is the wave speed in the medium. For the sake of simplicity, we will assume that on the scatterer Ω_0 the total field u satisfies homogeneous Dirichlet boundary conditions

$$u = 0|_{\partial\Omega_0}.$$

In electromagnetics, this condition corresponds to the case of scattering from a perfectly conducting obstacle in transverse-magnetic (TM) polarization, and in acoustics to the case of a sound-soft obstacle. Here and in what follows, $\boldsymbol{n} = (0, 1)$ is the unit normal on the line $y = 0$, $\frac{\partial u}{\partial n}$ denotes the partial derivative of u in the normal direction, and α is an impedance constant [9] with $\Re(\alpha) \geq 0$. The expression $[f]$ denotes the jump in the function f across the line $y = 0$, which we will denote by Γ in the remainder of the paper. We will denote by u^i the incoming field induced by the sources f in (1.1). We will limit our attention, without loss of generality, to either point sources or plane waves. To ensure uniqueness of the boundary value problem, a radiation condition must be imposed to enforce that the scattered field is decaying. Thus, we assume that the total field u is written in the form $u = u^i + u^s$, where the scattered field u^s satisfies the Sommerfeld radiation condition ([12], Chapter 2):

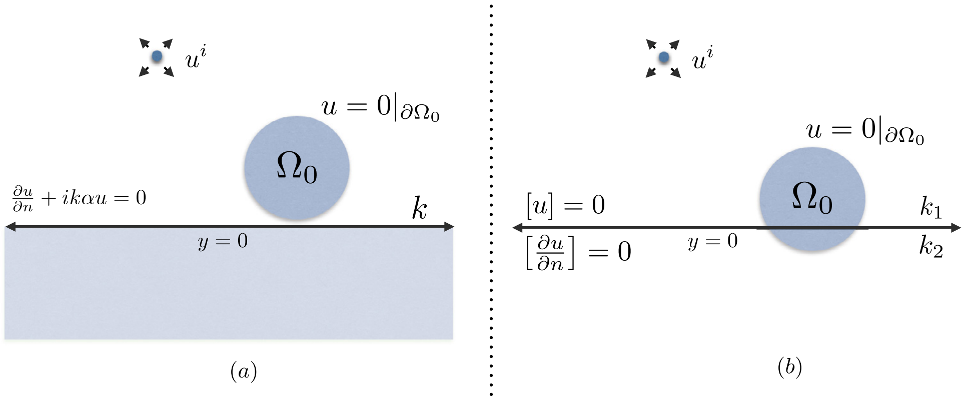

Fig. 1. (a) Scattering in the presence of an impedance half-space, with a point source defining the incoming field and a sound soft scatterer Ω_0. In (b), the impedance boundary is replaced by two-layer media, with a distinct Helmholtz parameter in the lower half-space. The scatterer Ω_0 is partially buried and touches both media.

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