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Theoretical analysis of the second-order synchrosqueezing transform

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ABSTRACT

We consider in this article the analysis of multicomponent signals, defined as superpositions of modulated waves also called modes. More precisely, we focus on the analysis of a variant of the second-order synchrosqueezing transform, which was introduced recently, to deal with modes containing strong frequency modulation. Before going into this analysis, we revisit the case where the modes are assumed to be with weak frequency modulation as in the seminal paper of Daubechies et al. [8], to show that the constraint on the compactness of the analysis window in the Fourier domain can be alleviated. We also explain why the hypotheses made on the modes making up the multicomponent signal must be different when one considers either wavelet or short-time Fourier transform-based synchrosqueezing. The rest of the paper is devoted to the theoretical analysis of the variant of the second order synchrosqueezing transform [16] and numerical simulations illustrate the performance of the latter.

1. Introduction

Multicomponent signals (MCs) are encountered in a number of fields of practical interest such as meteorology, structural stability analysis, and medical studies – see, e.g., [6,7,11,12]. Linear time–frequency (TF) analysis techniques have been extensively used to analyze and process these signals [14], the method to be used depending on the nature of the modes making up the signal. Standard linear TF methods such as the short-time Fourier transform (STFT) and the continuous wavelet transform (CWT) are commonly used to analyze such signals. In that context, the reassignment method (RM) was developed [1] to improve the readability of these linear TF representations. Unfortunately, since RM applies to the magnitude of the

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studied TF transform, the method results in a loss of information and the reassigned representation is not sufficient to recover the original signal.

Recently, Daubechies et al. [8] showed interesting results on the so-called synchrosqueezing transform (SST) to represent MCs, a method introduced in the mid-1990s for audio signal analysis [9]. SST combines the localization and sparsity properties of RM with the invertibility of a traditional TF representation. Originally proposed as a post-processing method applied to the CWT, SST can alternatively be applied to STFT with minor changes [15], to obtain the so-called FSST. Since the seminal paper of Daubechies et al. [8], many new developments have been carried out in various directions. First, an extension to the bidimensional case was proposed in [5], while a generalization of the wavelet approach by means of wavelet packets decomposition for both one dimensional and bidimensional cases is available in [26,25]. Finally, it is also worth noting that a study of synchrosqueezing applied to a more general class of multicomponent signals was done in [22].

In the present paper, we focus on what essentially limits the applicability of SST which are, on the one hand, the hypotheses of weak frequency modulation for the modes making up the signal and, on the other hand, the compactness of the frequency support of the analysis window. To better take into account the frequency modulation, in the FSST context, a first approach based on the definition of a demodulation operator was proposed in [13], while a new synchrosqueezing operator based on a new chirp rate estimate was defined in [16,2]. However, in these last two papers, no theoretical analysis of the proposed new synchrosqueezing transform was provided. To deal with this issue is the main aim of this paper.

While pursuing this goal, we will pay particular attention to use non-compactly supported window in the frequency domain. Indeed, the main problem induced by using windows compactly supported in the frequency domain is that they are not adapted to deal with real-time computations. To deal with this issue, a new wavelet-based SST defined using wavelet with sufficiently many vanishing moments and a minimum support in the time domain was proposed in [4], but the mathematical study of the corresponding instantaneous frequency (IF) estimates is still under development. In the meantime, an extension of synchrosqueezing based on wavelet packets not compactly supported in the frequency domain along with its mathematical analysis was developed in [24]. In the context of this paper, the focus is put on the Fourier-based SST, when the analysis window is not compactly supported. We first revisit the case of weak modulation for the modes using such a type of window, and then prove a new approximation theorem on a slightly different version of the synchrosqueezing transform proposed in [16].

The outline of the paper is as follows: in Section 2, we recall some notation and definitions. In Section 3, we introduce FSST and wavelet-based SST (WSST) along with the corresponding approximation results and explain why the hypotheses on the phase of the modes have to be different. Then, after having introduced some necessary ingredients, we derive, in Section 4, the approximation theorem for the second order synchrosqueezing transform, showing that this new transform is fully adapted for analyzing modes with strong frequency modulations. Finally, numerical illustrations conclude the paper.

2. Definitions

2.1. Short-time Fourier transform

We denote by $L^1(\mathbb{R})$ and $L^2(\mathbb{R})$ the space of integrable, and square integrable functions. Consider a signal $f \in L^1(\mathbb{R})$, and take a window g in the Schwartz class, $\mathcal{S}(\mathbb{R})$, the space of smooth functions with fast decaying derivatives of any order; its Fourier transform is defined by:

$$\widehat{f}(\eta) = \mathcal{F}\{f\}(\eta) = \int_{\mathbb{R}} f(\tau)e^{-2i\pi\eta\tau}d\tau. \quad (1)$$

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