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Letter to the Editor

A Fourier-invariant method for locating point-masses and computing their attributes

Charles K. Chui [a,1], H.N. Mhaskar [b,*,2]

[a] *Department of Mathematics, Hong Kong Baptist University, Hong Kong*
[b] *Institute of Mathematical Sciences, Claremont Graduate University, Claremont, CA 91711, United States*

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ABSTRACT

Motivated by the interest of observing the growth of cancer cells among normal living cells and exploring how galaxies and stars are truly formed, the objective of this paper is to introduce a rigorous and effective method for counting point-masses, determining their spatial locations, and computing their attributes. Based on computation of Hermite moments that are Fourier-invariant, our approach facilitates the processing of both spatial and Fourier data in any dimension.

1. Introduction

Throughout the history of mankind, people have always been fascinated by twinkling stars in the sky, and many have tried but failed to count them. The number of stars visible to the naked eye is estimated to be over 6,000, but one can see far less than half of them at the same time, even on a clear night, from either the northern or southern hemisphere. The question of how many stars in the universe has fascinated scientists, philosophers, poets, musicians, and dreamers throughout the ages. Ever since the invention of the telescope, commonly credited to the Dutch lens-maker, Hans Lippershey, in 1608, but significantly modified and improved by the "father of observational astronomy", Galileo Galilei, less than a year later, many types of powerful telescopes have been invented, particularly in the twentieth century, with the capability of observing celestial objects beyond the (human) visible light (of wavelengths from about 390 nm to 700 nm),

* Corresponding author.
E-mail addresses: ckchui@stanford.edu (C.K. Chui), hrushikesh.mhaskar@cgu.edu (H.N. Mhaskar).

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and ranging from the long radio wavelengths to the short wavelengths of gamma rays. However, though not being the very first, the launch of the Hubble space telescope (HST), on April 24, 1990, was surely a major breakthrough in (observational) astronomy. Indeed, HST allows scientists to observe stars (and even some of their planets and moons) in the electromagnetic spectrum that ranges between near-ultra violet and near-infrared, from outer space to avoid atmospheric turbulence effects that cause the "twinkling" visual and other artifacts, when observing stars from ground-based telescopes. While the HST orbits the Earth at an altitude of about 353 miles, the space telescope, Spitzer, launched in 2003, was designed to trail the Earth orbit and to scan the skies in infrared. The "image data" gathered by Hubble and Spitzer together, perhaps along with other space telescopes as well, allow us to have an approximate count of 70 billion trillion (or 7×10^{22}) stars and over 100 billion galaxies in the universe, as well as more than 300 billion stars in our own Milky Way (galaxy) alone.

The problem of observing, counting, isolating, and tracking (perhaps moving) point-masses is not limited to the investigation in astronomy. It is also important for research in biomedical sciences. For the heath-care sector, counting (red, white, and platelet) blood cells is routine in any regular physical examination. The importance of counting red blood cells (RBCs), known as erythrocyte count, is that the number of RBCs can affect the amount of oxygen the tissues receive, since RBCs contain hemoglobin which carries oxygen to the tissues. On the other hand, counting white blood cells (WBCs), along with their differentials (that is, breakdown of percentage of each WBC type) is for assessing the ability of the body to fight infection, since WBCs are produced and derived from multipoint cells in the bone marrow for protecting the body against both infectious disease and foreign invaders. The major function of platelets is to prevent bleeding, and its count is perhaps less important, since platelets are very small and make up only a tiny fraction of the blood volume anyway. The current practice in regular physical examination is to draw a test tube of blood from the patient for analysis in the laboratory. An alternative is to take a small blood sample, by drawing off a finger prick with a "Pasteur pipette", for immediate processing by an automatic counter. However, though less invasive, more timely, and low-cost, this alternative is usually not quite reliable. For non-invasive health care practice, it is clear that blood cell counting from digital imaging would be much more desirable. With the recent rapid advancement of biosensor and super-resolution optical microscopy technologies, it makes sense to invest more research effort in this direction. In this regard, various medical devices are already or soon to be available, and there are research papers, reports, and U.S. patents, such as [16,5,8,12,13,19,11,17], in the literature or web-pages. However, to the best our knowledge, there is no mathematically rigorous study in the current literature on counting RBCs and WBCs (along with their differentials) from imagery data without observation under the microscope.

The objective of this paper is to introduce a rigorous and effective method for counting, locating, and isolating point-masses along with their attributes, such as: galaxies and stars from spectral data acquired by space telescopes; red and white blood cells or other living cells from super-resolution optical microscopy; and other scientific applications. Before giving a precise formulation of the mathematical problem, let us first consider the ideal case, where the point-masses are represented by points (with zero measure), there is no noise nor other associated contamination or perturbation, and if applicable, the optical lens is an ideal thin lens that provides a Fourier transform optical observation tool. In other words, to motivate the statement of the mathematical problem, let the point-masses along with their attributes be represented by the counting measure:

$$\tau_I = \sum_{\ell=1}^{L} a_\ell \delta_{\mathbf{x}_\ell}, \tag{1.1}$$

where $\delta_{\mathbf{x}_\ell}$ denotes the Dirac delta at $\mathbf{x}_\ell$, and both the number of terms, L, in the summation, as well as the locations $\mathbf{x}_\ell$ along with their coefficients a_ℓ are unknown. Hence, an equivalent formulation of τ_I, via observation using the ideal thin lens, is given by:

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