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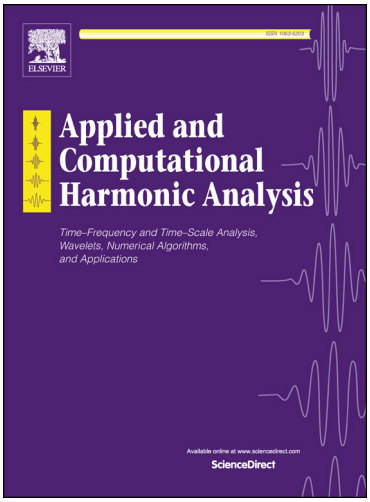

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Investigations of the effects of random sampling schemes on the stability of generalized sampling

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Abstract

Generalized sampling is a mathematical technique for obtaining approximations of signals with respect to different representations in a numerically stable manner. This can for example be relevant in processing MRI images, where hardware often enforces initial frequency measurements, but where a wavelet basis may be better suited for representing the image.

Recently the theory of generalized sampling was extended to work with arbitrary patterns in $\mathbb{R}^d$. In this article we investigate how the choice of the probability distribution generating random sampling schemes in $\mathbb{R}^2$ affects the numerical stability of generalized sampling.

Keywords: Generalized sampling, point processes, wavelets, numerical stability

1. Introduction

Generalized sampling [3, 2] is a technique for computing a representation of a function in one basis/frame of a Hilbert space from another. The theory is abstract and does not restrict the type of bases or frames that can be considered, and this provides the freedom to adapt the setup to the structure of the functions at hand. In a typical application, we have samples of the function given as inner products with respect to some fixed basis or frame imposed by the measuring process, but prior knowledge of the general function structure dictates a desire to change to another more efficient representation system for the function. For example, one may have access to Fourier samples of an image, but would like to change to a more efficient representation system for images such as wavelets.

An essential quantity ensuring numerical stability of the generalized sampling approach is the condition number of the change of basis (or frame) matrix (for short "condition number") between the two representation systems considered, see [2]. The condition number will, in general, depend on the sampling scheme and on the sampling and representation system. If the sampling scheme is a subset of a regular grid, the numerical stability of the change of basis matrix is well understood in the Fourier/wavelet setup

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