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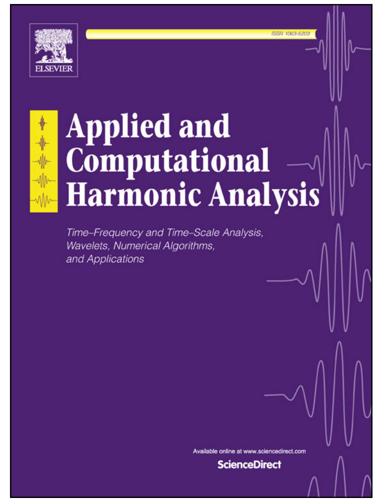

Discrete directional Gabor frames

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Discrete Directional Gabor Frames

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Abstract

We develop a theory of discrete directional Gabor frames for functions defined on the d-dimensional Euclidean space. Our construction incorporates the concept of ridge functions into the theory of isotropic Gabor systems, in order to develop an anisotropic Gabor system with strong directional sensitivity. We present sufficient conditions on a window function g and a sampling set Λ_ω for the corresponding directional Gabor system $\{g_{m,t,u}\}_{(m,t,u)\in\Lambda_\omega}$ to form a discrete frame. Explicit estimates on the frame bounds are developed. A numerical implementation of our scheme is also presented, and is shown to perform competitively in compression and denoising schemes against state-of-the-art multiscale and anisotropic methods, particularly for images with significant texture components.

1. Introduction

Given a square integrable function $g \in L^2(\mathbb{R})$ and constants $a, b > 0$, the associated *Gabor system* (also known as *Weyl–Heisenberg system*) generated by g and the lattice $a\mathbb{Z} \times b\mathbb{Z}$, $\mathcal{G}(g, a, b) = \{g_{m,n}\}_{m,n\in\mathbb{Z}}$, is defined by

$$g_{m,n}(x) = e^{2\pi iamx} g(x - bn).$$

In 1946, Dennis Gabor proposed to study such systems for their usefulness in the analysis of information conveyed by communication channels [1]. The resulting theory led to many applications ranging from auditory signal processing, to pseudodifferential operator analysis, to uncertainty principles. The edited volumes by Benedetto and Frazier [2] and by Feichtinger and Strohmer [3, 4], as well as Gröchenig's treatise [5], provide detailed treatments of various aspects of this rich and beautiful theory.

Among many interesting recent developments in time-frequency analysis, we want to consider the set of ideas which expands the notion of traditional Gabor systems by including directional information. Directional content is