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Second kind integral equation formulation for the mode calculation of optical waveguides

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A B S T R A C T

We present a second kind integral equation (SKIE) formulation for calculating the electromagnetic modes of optical waveguides, where the unknowns are only on material interfaces. The resulting numerical algorithm can handle optical waveguides with a large number of inclusions of arbitrary irregular cross section. It is capable of finding the bound, leaky, and complex modes for optical fibers and waveguides including photonic crystal fibers (PCF), dielectric fibers and waveguides. Most importantly, the formulation is well conditioned even in the case of nonsmooth geometries. Our method is highly accurate and thus can be used to calculate the propagation loss of the electromagnetic modes accurately, which provides the photonics industry a reliable tool for the design of more compact and efficient photonic devices. We illustrate and validate the performance of our method through extensive numerical studies and by comparison with semi-analytical results and previously published results.

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1. Introduction

Optical fibers and waveguides are important building blocks of many photonic devices and systems in telecommunication, data transfer and processing, and optical computing. Indeed, most photonic devices consist of approximately straight waveguides as input and output channels with complicated functional structures between the two. Two main mechanisms by which the electromagnetic wave can be confined in optical fibers or waveguides are total internal reflection and photonic band gap guidance [43,25]. Generally speaking, when the refractive index of the core is greater than that of the surrounding material, the light is confined in the core by total internal reflection; when the (hollow) core has a smaller refractive index,

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confinement can be achieved through photonic band gap guidance. In both cases, the propagating electromagnetic modes of optical fibers and waveguides depend on physical parameters such as the input light wavelength, refractive indices, and the geometry of the cross section of fibers and waveguides. To reduce the cost of designing new photonic devices, accurate and efficient simulation tools are in high demand in integrated photonics industry. The first step in the photonics simulation is to compute a complete set of propagating modes accurately and efficiently for optical fibers or waveguides.

There has been extensive research on the mode calculation of optical fibers and waveguides and various numerical methods have been developed. These include the effective index method [39,10], the plane wave expansion method [19,50,26], the multipole expansion method [56,57,33,12,11,55,31], finite difference methods [21,52,14,59], finite element methods [4,17,28,5,47,42,46], boundary integral methods [54,20,34,35,13,16,3,36,37,51,45], etc. Here we do not intend to review these methods in great detail, but note that the effective index method is generally of low order making it difficult to calculate the propagation constant to high accuracy; the plane wave expansion method implies an infinite periodic medium; the multipole expansion method requires that each core be of circular shape and that the cores be well separated from each other; finite difference and finite element methods requires a volume discretization of the cross section in a truncated computational domain with some artificial boundary conditions or perfectly matched layers imposed on or near the boundary of the truncated domain. When optical fibers and waveguides consist of many cores of arbitrary shape, these methods often need excessively large amount of computing resource in order to accurately calculate the imaginary part of the propagation constant, which is related to the propagation loss of the electromagnetic modes and thus of fundamental importance for the design purpose.

On the other hand, boundary integral methods represent the electromagnetic fields via layer potentials which satisfy the underlying partial differential equations automatically. One then derives a set of integral equations through the matching of boundary conditions with the unknowns only on the material interfaces. Thus the dimension of the problem is reduced by one and complex geometries can be handled relatively easily. Among the aforementioned work on boundary integral methods, [13,3,36,45] present numerical examples with high accuracy for smooth geometries. In [3] and [45], the field components E_z and H_z (with z-axis the longitudinal direction of the waveguide) are represented via four distinct *single* layer potentials and the resulting system is a mixture of first kind and singular integral equations; both authors apply the circular case as a preconditioner to obtain a well conditioned system for smooth boundaries. In [13], E_z and H_z are represented via a proper linear combination of *single* and *double* layer potentials in such a way that the hypersingular terms are canceled out. The resulting system still contains the tangential derivatives of the unknown densities and layer potentials and thus is not of the second kind. In [36], Dirichlet-to-Neumann (DtN) maps for H_x and H_y are used to construct a system of two integral equations, where each DtN map is in turn computed by a boundary integral equation with a hypersingular integral operator and a method in [30] is applied to evaluate the DtN map to high accuracy for smooth boundaries. While these methods are all capable of computing the propagation constant to high accuracy for smooth cases, it is not straightforward to extend them to treat nonsmooth cases such as standard dielectric rectangular waveguides in integrated optics.

Remark 1. We would like to remark here that [3] has a subsection titled "Buried Rectangular Dielectric Waveguide". In that subsection, the authors approximate the rectangular waveguide via a smooth super-ellipse and compute the propagation constant for the super-ellipse. Though Fig. 2 in [3] achieves about 9 digit accuracy for the super-ellipse, Fig. 3 in [3] shows only about two digit accuracy for the propagation constant of the genuine rectangular waveguide which is regarded as a limit of the super-ellipse.

In this paper, we construct a system of SKIEs formulation for the mode calculation of optical waveguides. Our starting point is the dual Müller's formulation [40] for the time-harmonic Maxwell's equations in three dimensions. We then reduce the dimension of the integration domain by one using the key assumption of