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Abstract

Wavelets on graphs have been studied for the past few years, and in particular, several approaches have been proposed to design wavelet transforms on hierarchical graphs. Although such methods are computationally efficient and easy to implement, their frames are highly restricted. In this paper, we propose a general framework for the design of wavelet transforms on hierarchical graphs. Our design is guaranteed to be a Parseval tight frame, which preserves the l_2 norm of any input signals. To demonstrate the potential usefulness of our approach, we perform several experiments, in which we learn a wavelet frame based on our framework, and show, in inpainting experiments, that it performs better than a Haar-like hierarchical wavelet transform and a learned treelet. We also show with category theory that the algebraic properties of the proposed transform have a strong relationship with those of the hierarchical graph that represents the structure of the given data.

Keywords: Wavelet transform, directed acyclic graph, Parseval tight frame, category theory

1. Introduction

Interest in signal processing algorithms in various applications has increased in recent years. Examples of these signal processing algorithms include sensor networks, transportation networks, power grids, the Internet, and social networks. In these applications, data are defined on topologically complicated domains, such as high-dimensional structures, irregularly sampled spaces, and nonlinear manifolds. Graphs are effective ways to represent the geometric structures of data on complicated domains. Several attempts have been made to extend conventional signal processing techniques to graph signals. For example, an extension of the traditional digital signal processing, such as filters, impulse response, and z-transform, has been proposed [1]. Graph Fourier transforms, which provide a harmonic analysis of graph signals based on the graph Laplacian [2], have been proposed as well [3, 4, 5].

The wavelet transform is a powerful conventional signal processing tool for one- and two-dimensional (1D and 2D) discrete signals, and it has been widely applied in several fields, including signal analysis, data compression, and denoising [6]. The wavelet transform provides a sparse representation of piecewise smooth signals, and thus it has the ability to effectively represent a wide class of signals. Recently, the possibility of extending the wavelet transform to data on graphs has been investigated. Wavelet transforms based on the spectral graph theory, which are applicable to data on a weighted undirected graph, have been considered [7, 8, 9]. These techniques assume that nodes represent data samples with signals that we want to process. In this representation, similar samples are connected by an edge, and the similarity between the two samples is often represented by the weight associated with the edge.

In another approach, wavelet transforms for data on hierarchical graphs have been proposed [10, 11, 12, 13, 14]. The main advantages of these methods are that they are computationally efficient and easy to implement. Hierarchical graphs are also effective ways to represent the data structures on complicated domains. In this representation, data samples with signals are often represented by leaves (i.e., nodes without children), and similar nodes have the same parent. It should be emphasized that the representation of the data structure by a hierarchical graph can be interpreted by the hierarchical clustering, where each non-leaf node corresponds to a cluster containing its children. A tree,

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