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Random sampling of bandlimited signals on graphs☆

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Abstract

We study the problem of sampling k-bandlimited signals on graphs. We propose two sampling strategies that consist in selecting a small subset of nodes at random. The first strategy is non-adaptive, *i.e.*, independent of the graph structure, and its performance depends on a parameter called the graph coherence. On the contrary, the second strategy is adaptive but yields optimal results. Indeed, no more than $O(k \log(k))$ measurements are sufficient to ensure an accurate and stable recovery of all k-bandlimited signals. This second strategy is based on a careful choice of the sampling distribution, which can be estimated quickly. Then, we propose a computationally efficient decoder to reconstruct k-bandlimited signals from their samples. We prove that it yields accurate reconstructions and that it is also stable to noise. Finally, we conduct several experiments to test these techniques.

1. Introduction

Graphs are a central modelling tool for network-structured data [1]. Depending on the application, the nodes of a graph may represent people in social networks, brain regions in neuronal networks, or stations in transportation networks. Data on a graph, such as individual hobbies, activity of brain regions, traffic at a station, may be represented by scalars defined on each node, which form a graph signal. Extending classical signal processing methods to graph signals is the purpose of the emerging field of graph signal processing [2, 3].

Within this framework, a cornerstone is sampling, *i.e.*, measuring a graph signal on a reduced set of nodes carefully chosen to enable stable reconstructions. Classically, sampling a continuous signal $x(t)$ consists in measuring a countable sequence of its values, $\{x(t_j)\}_{j\in\mathbb{Z}}$, that ensures its recovery under a given smoothness model [4]. Smoothness assumptions are often defined in terms of the signal's Fourier transform. For example, Shannon's famous sampling theorem [5] states that any ω-bandlimited signal can be

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