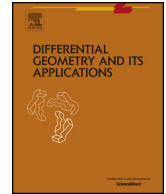




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The conformal vector fields on Kropina manifolds

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ABSTRACT

In this paper we study and characterize the conformal vector fields on Kropina manifolds. Furthermore, we obtain the explicit expressions of conformal vector fields on a Kropina manifold (M, F) of weakly isotropic flag curvature with the conditions that $b = \|\beta\|_\alpha$ is a constant and the dimension of M is greater than two.

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1. Introduction

There is a special and important class of Finsler metrics in Finsler geometry which can be expressed in the form $F = \alpha\phi(\beta/\alpha)$, where α is a Riemannian metric and β is a 1-form and $\phi = \phi(s)$ is a C^∞ positive function on an open interval. We call this class of metrics the (α, β) -metrics. In the past several years, we witness a rapid development in Finsler geometry. This is partially due to the study on (α, β) -metrics ([2]). When $\phi = 1 + s$, the Finsler metric $F = \alpha + \beta$ is called *Randers metric*. When $\phi = 1/s$, the Finsler metric $F = \alpha^2/\beta$ is called *Kropina metric*. Randers metrics and Kropina metrics are both C-reducible ([10]). However, Randers metrics are regular Finsler metrics but Kropina metrics are Finsler metrics with singularity. Kropina metrics were first introduced by L. Berwald when he studied the two-dimensional Finsler spaces with rectilinear extremal and were investigated by V.K. Kropina (see [7][8]). Kropina metrics seem

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to be among the simplest nontrivial Finsler metrics with many interesting application in physics, electron optics with a magnetic field, dissipative mechanics and irreversible thermodynamics (see [1][6]). Also, they have interesting applications in relativistic field theory, control theory, evolution and developmental biology.

In this paper, we mainly study a class of Kropina metrics which are regular Finsler metrics on a manifold M . At each $x \in M$, we define

$$A_x = \left\{ y = y^i \frac{\partial}{\partial x^i} \in T_x M \mid \beta = b_i(x)y^i > 0 \right\},$$

which is a conic domain of $T_x M$ whose boundary is the hyperplane $\{y = y^i \frac{\partial}{\partial x^i} \in T_x M \mid b_i(x)y^i = 0\}$. The Kropina metric $F = \alpha^2/\beta: A \rightarrow (0, \infty)$ is a conic Finsler metric which is positive definite Finsler metric on A , where $A := \bigcup_{x \in M} A_x$ (see [16]). This class of metrics are called the *conic Kropina metrics*. In the following, we just study conic Kropina metrics and we always use Kropina metric to take the place of conic Kropina metric.

The flag curvature in Finsler geometry is a natural analogue of sectional curvature in Riemannian geometry and is an important Riemannian geometric quantity. For a Finsler manifold (M, F) , the flag curvature $K = K(P, y)$ of F is a function of “flag” $P \in T_x M$ and “flagpole” $y \in T_x M$ at x with $y \in P$. A Finsler metric F is said to be of *weakly isotropic flag curvature* if $K = \frac{3\theta}{F} + \sigma$, where $\sigma = \sigma(x)$ is a scalar function and θ is a 1-form on M . In 1978, C. Shibata studied some basic local geometrical properties of Kropina spaces (see [11]). In 1991, M. Matsumoto obtained a set of necessary and sufficient conditions for a Kropina manifold to be of constant flag curvature (see [9]). Based on these results, R. Yoshikawa and K. Okubo characterized a Kropina metric $F = \frac{\alpha^2}{\beta}$ via navigation data (h, W) , where h is a Riemannian metric and W is a unit length vector field, and obtained a minimal set of necessary and sufficient local conditions for a Kropina space to be of constant flag curvature (see [14,15]). Recently, Q. Xia has classified regular Kropina metrics of weakly isotropic flag curvature on the manifolds with the dimension greater than two under the condition that $b = \|\beta\|_\alpha$ is a constant (see [13]).

The conformal vector fields are the fundamental objects in differential geometry and physics. The conformal vector fields on a manifold M are vector fields induced by a local one-parameter group of conformal transformations of M . For Randers manifolds, based on the classification theorem on Randers metrics of weakly isotropic flag curvature given by the first author and Z. Shen in [3], Z. Shen and Q. Xia completely determined conformal vector fields on Randers manifolds of weakly isotropic flag curvature (see [12]). Then a natural problem arises: how to determine the conformal vector fields on Kropina manifolds of weakly isotropic flag curvature? In order to solve this problem, we first consider conformal vector fields on a Kropina manifold and obtain the following theorem.

Theorem 1.1. *Let $F = \frac{\alpha^2}{\beta}$ be a Kropina metric on a smooth manifold M . Then a vector field V on (M, F) is a conformal vector field with conformal factor $\rho = \rho(x)$ if and only if V satisfies the following conditions:*

$$V_{i;j} + V_{j;i} = 4\lambda a_{ij}, \tag{1.1}$$

$$V^i b_{j;i} + b^i V_{i;j} = 2(2\lambda - \rho)b_j, \tag{1.2}$$

where we raise and lower the indices of V by a_{ij} , “;” denotes the covariant derivative with respect to Levi-Civita connection of Riemannian metric α and $\lambda = \lambda(x)$ is a scalar function on M .

Further, we will completely determine conformal vector fields from the PDEs in the [Theorem 1.1](#) when F is a Kropina metric of weakly isotropic flag curvature and $b := \|\beta\|_\alpha$ is a constant.

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