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Determining Projection Constants of Univariate Polynomial Spaces

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Abstract

The long-standing problem of minimal projections is addressed from a computational point of view. Techniques to determine bounds on the projection constants of univariate polynomial spaces are presented. The upper bound, produced by a linear program, and the lower bound, produced by a semidefinite program exploiting the method of moments, are often close enough to deduce the projection constant with reasonable accuracy. The implementation of these programs makes it possible to find the projection constant of several three-dimensional spaces with five digits of accuracy, as well as the projection constants of the spaces of cubic, quartic, and quintic polynomials with four digits of accuracy. Beliefs about uniqueness and shape-preservation of minimal projections are contested along the way.

Key words and phrases: Minimal projection, projection constant, linear programming, semidefinite programming, method of moments.

AMS classification: 41A44, 65K05, 90C22, 90C47.

1 Introduction

The problem of minimal projections has attracted the attention of approximation theorists for about half a century. The survey of Cheney and Price [3] still provides a fine account on the topic. The fact that the Fourier projection is uniquely minimal from $\mathcal{C}(\mathbb{T})$ onto the space of trigonometric polynomials of degree at most d, derived from Berman–Marcinkiewicz formula, undoubtedly stands as a highlight of the subject, see [12, 4]. But when the focus is put on algebraic rather than trigonometric polynomials, the situation becomes dramatically more complicated. Besides the trivial cases of degree $d = 0$ and $d = 1$, only the case $d = 2$ has been resolved, albeit at the cost of considerable efforts deployed by Chalmers and Metcalf [2]. In fact, traditional analyses may have

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