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PII: S0021-9045(18)30043-1
DOI: https://doi.org/10.1016/j.jat.2018.04.013
Reference: YJATH 5218

To appear in: *Journal of Approximation Theory*

Received date : 1 February 2017
Revised date : 18 April 2018
Accepted date : 23 April 2018

Please cite this article as: A.G. Georgiadis, M. Nielsen, Spectral multipliers on spaces of distributions associated with non-negative self-adjoint operators, *Journal of Approximation Theory* (2018), https://doi.org/10.1016/j.jat.2018.04.013

SPECTRAL MULTIPLIERS ON SPACES OF DISTRIBUTIONS ASSOCIATED WITH NON-NEGATIVE SELF-ADJOINT OPERATORS

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Abstract. We consider spaces of homogeneous type associated with a non-negative self-adjoint operator whose heat kernel satisfies certain upper Gaussian bounds. Spectral multipliers are introduced and studied on distributions associated with this operator. The boundedness of spectral multipliers on Besov and Triebel-Lizorkin spaces with full range of indices is established too. As an application, we obtain equivalent norm characterizations for the spaces mentioned above. Non-classical spaces as well as Lebesgue, Hardy, (generalized) Sobolev and Lipschitz spaces are also covered by our approach.

1. Introduction

Spaces of functions and distributions play a prominent role in several areas of modern mathematics, such as complex analysis, harmonic analysis and the theory of partial differential equations. There is also a strong connection to applicable mathematics in areas such as approximation theory [25, 26, 29], as well as in probability theory, statistics, signal theory and image processing [9, 14, 27]. Some of the most general scales of distributions are Triebel-Lizorkin and Besov spaces (see for example [3–6, 11, 12, 17, 18, 20, 26, 30, 35, 36, 38]), measuring simultaneously smoothness and integrability of distributions. They in turn include many classical functions spaces like Lebesgue, Hardy, Sobolev, BMO and Hölder-Zygmund spaces.

The focus in harmonic analysis has broadened from the study of objects associated with Euclidean space to more general settings based on Lie groups, manifolds and metric measure spaces in general. Distributions on metric measure spaces associated with operators were developed recently by Kerkyacharian and Petrushev in [27]. Moreover, spaces of distributions, and especially Besov $B^s_{pq}(L)$ and Triebel-Lizorkin $F^s_{pq}(L)$ spaces, associated with a non-negative self-adjoint operators L have been studied in [11–13, 21, 27, 30].

One of the classical and important problems related to spaces distribution is the question of boundedness of Fourier multipliers [24,32,34]. The Fourier transform on $\mathbb{R}^n$ provides a very natural setting for this problem. Moreover, the setting can easily be extended by replacing Fourier theory by more general spectral theory. Multiplier operators of this type are called spectral multipliers and are well-developed in the

2010 *Mathematics Subject Classification.* Primary: 58J35, 58J40. Secondary: 43A15, 43A85, 46E35, 46F05.

Key words and phrases. Besov spaces, distributions, doubling property, equivalent norms, Hardy spaces, Lipschitz spaces, self-adjoint operators, Sobolev spaces, spaces of homogeneous type, spectral multipliers, Triebel-Lizorkin spaces.

Supported by the Danish Council for Independent Research — Natural Sciences, grant 12-124675, "Mathematical and Statistical Analysis of Spatial Data".