Periodic perturbations of unbounded Jacobi matrices III: The soft edge regime

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PERIODIC PERTURBATIONS OF UNBOUNDED JACOBI MATRICES III: THE SOFT EDGE REGIME

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Abstract. We present pretty detailed spectral analysis of Jacobi matrices with periodically modulated entries in the case when 0 lies on the soft edge of the spectrum of the corresponding periodic Jacobi matrix. In particular, we show that the studied operators are always self-adjoint irrespective of the modulated sequence. Moreover, if the growth of the modulated sequence is superlinear, then the spectrum of the considered operators is always discrete. Finally, we study regular perturbations of this class in the linear and the sublinear cases. We impose conditions assuring that the spectrum is absolute continuous on some regions of the real line. A constructive formula for the density in terms of Turán determinants is also provided.

1. Introduction

Consider two sequences $a = (a_n : n \geq 0)$ and $b = (b_n : n \geq 0)$ such that for every $n \geq 0$ one has $a_n > 0$ and $b_n \in \mathbb{R}$. Then one defines the symmetric tridiagonal matrix by the formula

$$\mathcal{A} = \begin{pmatrix} b_0 & a_0 & 0 & 0 & \ldots \\ a_0 & b_1 & a_1 & 0 & \ldots \\ 0 & a_1 & b_2 & a_2 & \ldots \\ 0 & 0 & a_2 & b_3 & \\ \vdots & \vdots & \vdots & & \ddots \end{pmatrix}.$$

The action of $\mathcal{A}$ on any sequence is defined by the formal matrix multiplication. Let the operator A be the restriction of $\mathcal{A}$ to ℓ^2, i.e. $\operatorname{Dom}(A) = \{x \in \ell^2 : \mathcal{A}x \in \ell^2\}$ and $Ax = \mathcal{A}x$ for $x \in \operatorname{Dom}(A)$, where

$$\langle x, y \rangle_{\ell^2} = \sum_{n=0}^{\infty} x_n \overline{y_n}, \quad \ell^2 = \{x \in \mathbb{C}^{\mathbb{N}} : \langle x, x \rangle_{\ell^2} < \infty\}.$$

The operator A is called *Jacobi matrix*. It is self-adjoint provided Carleman condition is satisfied, i.e.

(1) $$\sum_{n=0}^{\infty} \frac{1}{a_n} = \infty.$$

A generalised eigenvector u associated with $x \in \mathbb{R}$ is any sequence satisfying the recurrence relation

(2) $$a_{n-1} u_{n-1} + b_n u_n + a_n u_{n+1} = x u_n \quad (n \geq 1).$$

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