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SIMULTANEOUS UNIVERSALITY

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Abstract. In this paper, the notion of simultaneous universality is introduced, concerning operators having orbits that simultaneously approximate any given vector. This notion is related to the well known concepts of universality and disjoint universality. Several criteria are provided, and several applications to specific operators or sequences of operators are performed, mainly in the setting of sequence spaces or spaces of holomorphic functions.

1. Introduction

In this paper, we are concerned with the phenomenon of simultaneous approximation by the action of several operators or, more generally, by the action of several sequences of mappings. When the existence of a dense orbit under an operator is proved, we are speaking about universality or hypercyclicity, see below. In many situations, it is possible to show the existence of one vector whose orbits under two or more operators approximate any given vector. Pushing the question quite further, we wonder under what conditions such approximation takes place by using a *common* subsequence. This, together with its connection with other kinds of joint universality, will make up the main aim of the present manuscript.

Next, we fix some related notation and terminology to be used in this work. For a good account of concepts, results and history concerning hypercyclicity, the reader is referred to the books [2, 21].

By $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{D}$, $B(a,r)$, $\overline{B}(a,r)$ ($a \in \mathbb{C}$, $r > 0$) we denote, respectively, the set of positive integers, the set $\mathbb{N} \cup \{0\}$, the real line, the complex plane, the open unit disk $\{z \in \mathbb{C} : |z| < 1\}$, the open disk with center a and radius r, and the corresponding closed disk. Let X, Y be two Hausdorff topological spaces, and $T_n : X \to Y$ ($n = 1, 2, \dots$) be a sequence of continuous mappings. Recall that (T_n) is said to be *universal* whenever there is some (T_n)-orbit which is dense in Y, that is, there exists an element $x_0 \in X$ –called universal for (T_n)– such that

$$\overline{\{T_n x_0 : n \in \mathbb{N}\}} = Y.$$

Note that Y must be separable. We denote by $\mathcal{U}((T_n))$ the set of universal elements for (T_n). When $X = Y$ and $T : X \to X$ is a continuous self-mapping,

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