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THE ESSENTIAL SPECTRUM OF SOME UNBOUNDED JACOBI MATRICES: A GENERALIZATION OF THE LAST–SIMON APPROACH

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Abstract. We consider some unbounded Jacobi matrices $\mathcal{J}$ with zero main diagonal and off diagonal entries defined by two different rules and calculate their essential spectrum by extending Last and Simon's ideas from the bounded to the unbounded case. We show that the essential spectrum of $\mathcal{J}$ is the union of the spectra of three *limit matrices* $\mathcal{J}_z$, $\mathcal{J}_{zc}$, and $\mathcal{J}_{cz}$. Finally, we give a description of each of these spectra.

1. Introduction

For a given sequence $\{a_k\}_1^\infty$ of positive numbers and a sequence $\{b_k\}_1^\infty$ of real numbers the Jacobi matrix (operator) $\mathcal{J}$ acts in the space F of sequences with a finite number of nonzero components $f = \{f_k\}_1^\infty$ by the formula

$$(\mathcal{J}f)_k = a_{k-1}f_{k-1} + b_k f_k + a_k f_{k+1},$$

where $k = 1, 2, \dots$ and $a_0 = f_0 = 0$ by definition. The numbers a_k are called the "weights" of $\mathcal{J}$. In what follows we consider only Jacobi matrices with zero diagonal entries $b_k \equiv 0$. One could consider more general classes of Jacobi matrices with non-trivial main diagonal but it would lead to longer calculations but not to better clarity of ideas. Below we will consider the minimal operator $\mathcal{J}$ which is the closure of $\mathcal{J}|_F$ in the Hilbert space $\ell^2(\mathbb{N})$.

In this work the essential spectrum $\sigma_{\text{ess}}(\mathcal{J})$ of a special class of unbounded Jacobi matrices $\mathcal{J}$ with weights given by two different rules is calculated. This non-homogeneous type of weights makes the analysis of the essential spectrum of $\mathcal{J}$ non-trivial and interesting. The calculation of $\sigma_{\text{ess}}(\mathcal{J})$ is based on the ideas of Last–Simon from [12] described by them in the case of arbitrary bounded Jacobi matrices. Although the calculation of the essential spectra by using the "limit matrices" has a rich story [13, 14] our technique refers mainly to [3].

As it has been mentioned in [12] in the case of unbounded Jacobi matrices this approach encounters essential difficulties. Indeed, the calculation of the essential spectra based on the analysis of spectra of the "limit matrices", which is valid in the case of bounded Jacobi matrices fails for general unbounded ones. This can be easily seen even in a rather simple case.

Consider the selfadjoint unbounded Jacobi matrix $\mathcal{J}$ given by the choice

$$a_n = n^\alpha \text{ and } b_n = 0, \quad n \in \mathbb{N}, \quad 0 < \alpha < 1.$$

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