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Higher Markov and Bernstein inequalities and fast decreasing polynomials with prescribed zeros

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Abstract

Higher order Bernstein- and Markov-type inequalities are established for trigonometric polynomials on compact subsets of the real line and for algebraic polynomials on compact subsets of the unit circle. In the case of Markov-type inequalities we assume that the compact set satisfies an interval condition.
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1. Introduction

Two of the most classical polynomial inequalities are the Bernstein inequality (see [2], p. 233 Theorem 5.1.7 or [14], p. 532, Theorem 1.2.5)

$$\left|P_n'(x)\right| \leq \frac{n}{\sqrt{1-x^2}}\|P_n\|_{[-1,1]}, \quad x \in (-1,1),$$

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and the Markov inequality (see [2], p. 233 Theorem 5.1.8 or [14], p. 529 Theorem 1.2.1)

$$\|P_n'\|_{[-1,1]} \le n^2 \|P_n\|_{[-1,1]},$$

where P_n is an algebraic polynomial of degree at most n, and $\|\cdot\|_X$ denotes the sup-norm over the set X. For any trigonometric polynomial T_n of degree at most n the following Bernstein-type inequality holds (established by M. Riesz, see [14], p. 532 Theorem 1.2.4 or [2], p. 232 Theorem 5.1.4)

$$\|T_n'\|_{[0,2\pi]} \le n\|T_n\|_{[0,2\pi]}.$$

There is also an analogue of this inequality for trigonometric polynomials on an interval less than the period see [2] p. 243. In 2001, Vilmos Totik developed the method of polynomial inverse images to prove asymptotically sharp Bernstein- and Markov-type inequalities for algebraic polynomials on several intervals [24], and in [27] asymptotically sharp inequalities were also obtained for trigonometric polynomials on several intervals and for algebraic polynomials on several circular arcs on the complex plane. The case of one circular arc was considered earlier in [16]. In the recently published paper [7] algebraic polynomials on sets satisfying (2) were considered, for trigonometric polynomials, see [6]. The next step in generalization of this result was done in [29], where asymptotic higher order Markov-type inequalities for algebraic polynomials on compact sets satisfying (2) were established.

The purpose of the present paper is to extend these results to trigonometric polynomials and to algebraic polynomials on subsets of the unit circle and to present a new type of fast decreasing polynomials. Briefly, the approach of Vilmos Totik and Yuan Zhou [29] was to establish the Markov-type inequality for T-sets, then for general sets and use Faà di Bruno's formula and Remez inequality near interior critical points. The difference here is that we developed fast decreasing polynomials with prescribed zeros to deal with interior critical points. Moreover, we also establish higher order Bernstein-type inequalities.

Sharp higher order Markov-type inequality is established for sets satisfying the interval condition (2). At interior points sharp Bernstein-type inequality is also derived which involves much slower growth order ($O(n^{2k})$ at endpoints vs. $O(n^k)$ at interior points where kth derivatives are considered).

The structure of the paper is the following. First, notation is introduced, and some known, basic results about T-sets are mentioned. Then the important density results (for T-sets and regular sets) are recalled. New results are in Section 3. A construction of fast decreasing polynomials with prescribed zeros can also be found here. A preliminary, "rough" Markov- and Bernstein-type inequalities are needed for special sets. Then the asymptotically sharp Markov-type inequality is formulated for higher derivatives of trigonometric polynomials and for algebraic polynomials on subsets of the unit circle. Finally, asymptotically sharp Bernstein-type inequalities are established in the trigonometric case as well as in the algebraic case.

2. Notation, background

We denote by $\mathbf{R}$ the real line, by $\mathbf{C}$ the complex plane, by $\overline{\mathbf{C}}$ the extended complex plane, and by $\mathbb{T}$ the unit circle and by $\mathbf{N}$ the nonnegative integers.

We use Faà di Bruno's formula (or Arbogast's formula; see [9], p. 17 or [21], pp. 35–37 or [5]): if f and g are k times differentiable functions, then

$$\frac{d^k}{dx^k} f(g(x)) = \sum \frac{k!}{m_1!m_2!\dots m_k!} f^{(m_1+m_2+\cdots+m_k)}(g(x)) \prod_{j=1}^{k} \left(\frac{g^{(j)}(x)}{j!}\right)^{m_j} \tag{1}$$

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