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On the universal functions.

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Abstract

In this article a function $U \in L^1([-\pi,\pi])$ and a set $E \subset [-\pi,\pi]$ with $0 < \mu E < 2\pi$ are constructed, such that for each function $f \in L^1(E)$ one can find a sequence $\{\delta_k = \pm 1\}_{k=1}^{\infty}$ and a function $g \in L^1([-\pi,\pi])$ which coincides with $f(x)$ on E and satisfies $c_k(g) = \delta_k c_k(U)$ $(k = 0, \pm 1, \pm 2, ...)$, where $\{c_k(g)\}_{k\in Z}$ and $\{c_k(U)\}_{k\in Z}$ are the Fourier coefficients of functions g and U, respectively.

Keywords: Trigonometric Fourier series; Fourier coefficients; Rudin-Shapiro polynomials; Universal pair

2010 MSC: 42A16, 42B05

1. Introduction

Let E be a Lebesgue measurable subset of $T = [-\pi,\pi]$. By $L^1(E)$ we denote the space of all Lebesgue integrable functions on E. Let

$$c_k(f) = \frac{1}{2\pi}\int_T f(x)e^{-ikx}dx \quad (k = 0, \pm 1, \pm 2, ...)$$

be the complex Fourier coefficients of $f \in L^1(T)$ with respect to the complex trigonometric system $\{e^{ikx}\}_{k\in Z}$. We denote the nth partial sum of the Fourier series of a function $f \in L^1(T)$ by $S_n(x,f)$, that is,

$$S_n(x,f) = \sum_{k=-n}^{n} c_k(f)e^{ikx} \quad (n = 0, 1, 2, ...).$$

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