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Mixed type multiple orthogonal polynomials associated with the modified Bessel functions and products of two coupled random matrices

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Abstract

We consider mixed type multiple orthogonal polynomials associated with a system of weight functions consisting of two vectors. One vector is defined in terms of scaled modified Bessel function of the first kind I_μ and $I_{\mu+1}$, the other vector is defined in terms of scaled modified Bessel function of the second kind K_ν and $K_{\nu+1}$. We show that the corresponding mixed type multiple orthogonal polynomials exist. For the special case that each multi-index is on or close to the diagonal, basic properties of the polynomials and their linear forms are investigated, which include explicit formulas, integral representations, differential properties, limiting forms and recurrence relations. It comes out that, for specified parameters, the linear forms of these mixed type multiple orthogonal polynomials can be interpreted as biorthogonal functions encountering in recent studies of products of two coupled random matrices. This particularly implies a Riemann-Hilbert characterization of the correlation kernel, which provides an alternative way for further asymptotic analysis.

Keywords: mixed type multiple orthogonal polynomials, modified Bessel functions, integral representations, limiting forms, recurrence relations, random matrices

1 Introduction

Multiple orthogonal polynomials are polynomials of one variable which are defined by orthogonality conditions with respect to several different weights. The general definition requires two sets of functions defined on the real axis $\mathbb{R}$:

$$\mathbf{w}_1 = (w_{1,1}, \dots, w_{1,p}), \qquad \mathbf{w}_2 = (w_{2,1}, \dots, w_{2,q}),$$

where $p, q \in \mathbb{N} = \{1, 2, 3, \dots\}$ and two multi-indices

$$\mathbf{n}_1 = (n_{1,1}, \dots, n_{1,p}) \in \mathbb{Z}_+^p, \qquad \mathbf{n}_2 = (n_{2,1}, \dots, n_{2,q}) \in \mathbb{Z}_+^q,$$

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